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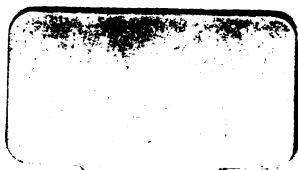


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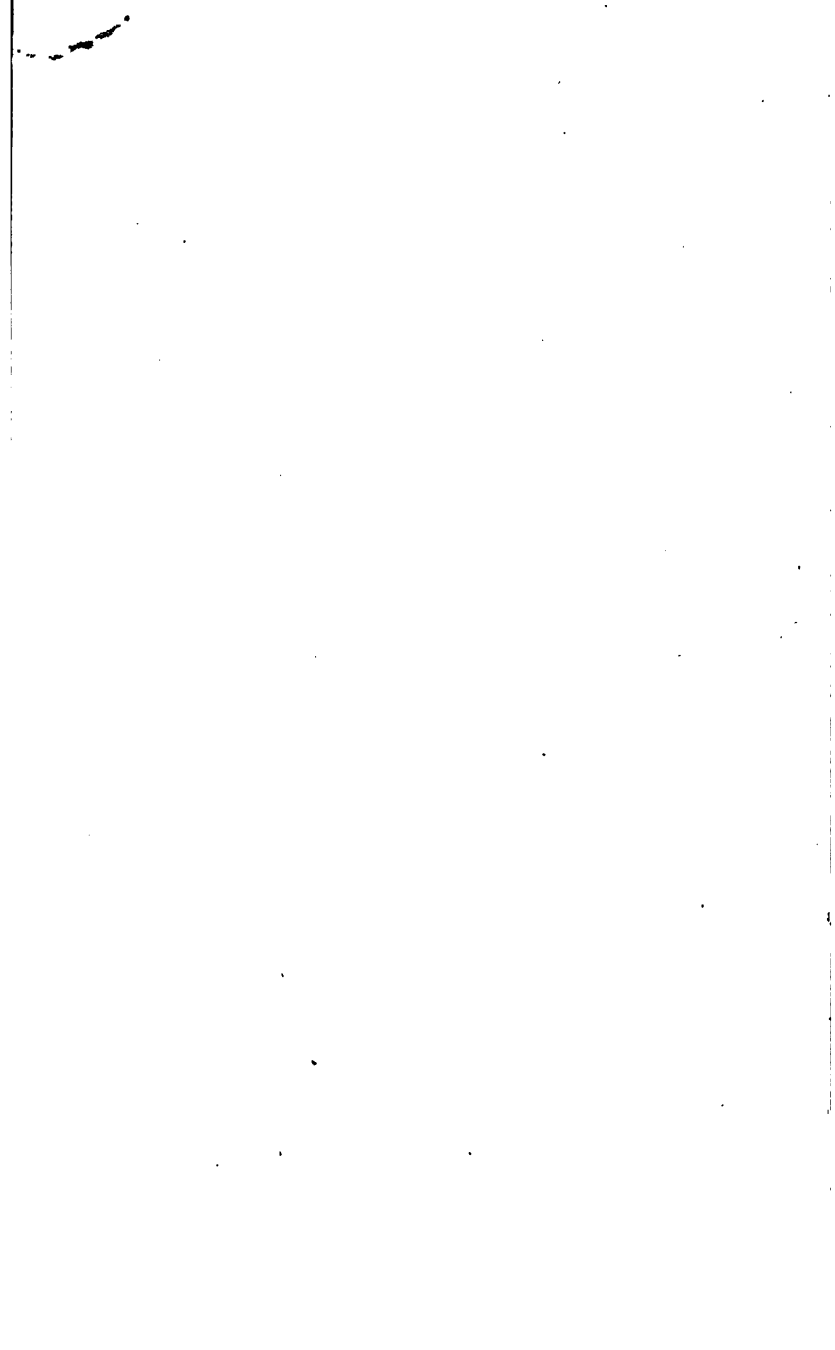


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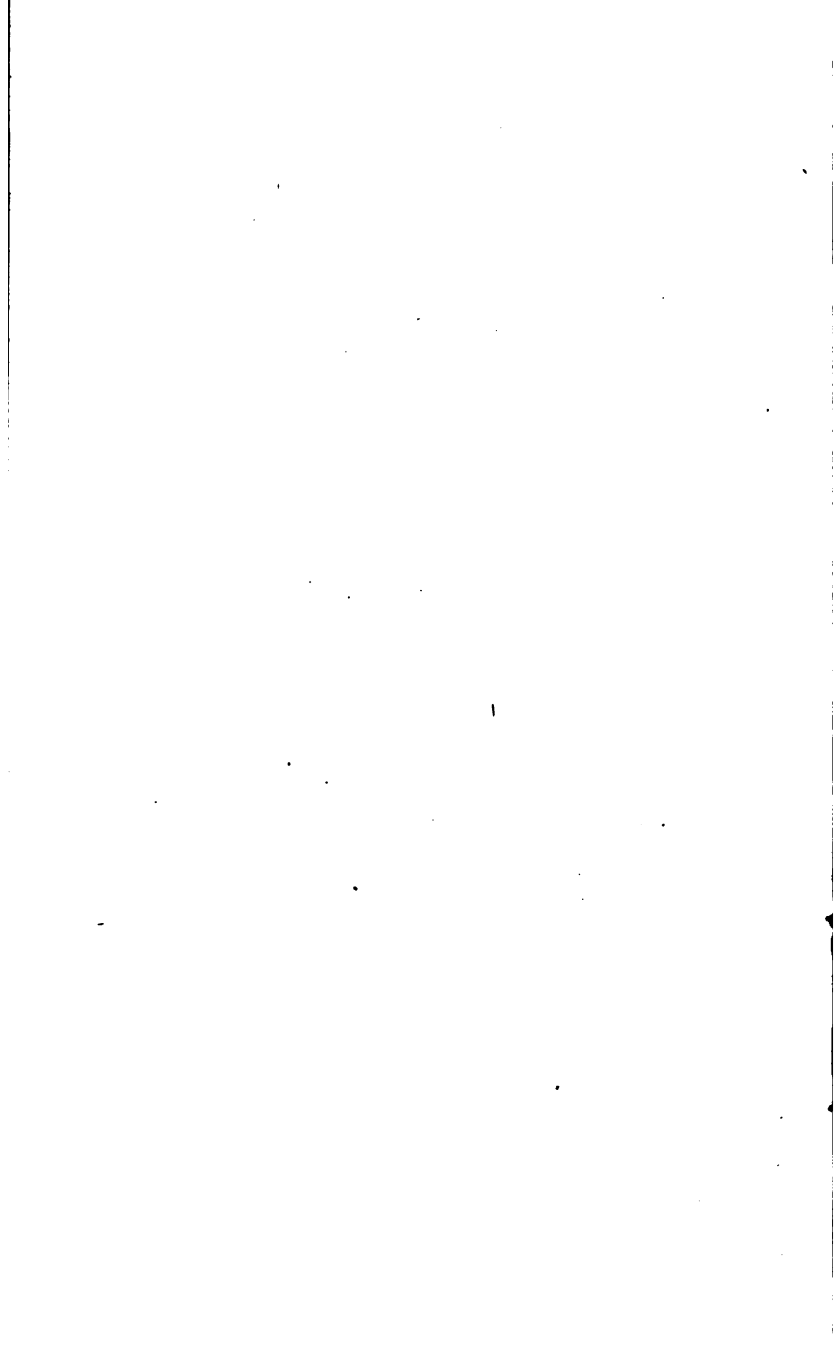
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A
KEY
TO THE
FIRST LESSONS IN ALGEBRA,
CONTAINING THE
ANSWERS TO THE QUESTIONS,
WITH NUMEROUS
EXPLANATIONS AND SOLUTIONS.

BY EBENEZER BAILEY,
PRINCIPAL OF THE YOUNG LADIES' HIGH SCHOOL, BOSTON; AUTHOR OF
"YOUNG LADIES' CLASS BOOK," &c.



BOSTON:
JENKS AND PALMER.

1841.

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Jul. 27, 1916

Gift of

Mrs. G. L. Batchelder
Medford.

At a meeting of the School Committee of the City of
Boston, March 11, 1834,

Voted, That "BAILEY'S ALGEBRA" be used in the
Writing Schools in which Algebra is allowed to be taught.

Attest,

S. F. McCLEARY, *Secretary*

Entered according to Act of Congress, in the year 1834,

By EBENEZER BAILEY,

In the Clerk's Office of the District Court of Massachusetts.

KEY

TO

BAILEY'S ALGEBRA.

CHAPTER II.

ADDITION

SECTION I.

- | | |
|-------------------------------------|----------------------------|
| 1, 2, 3. Answers given in the book. | 16. $\sim 40 a b c d.$ |
| 4. $14 a b c.$ | 17. $\sim 17 a b x.$ |
| 5. $16 y z.$ | 18. $\sim 39 a b x y.$ |
| 6. $9 a x.$ | 19. $\sim 38 a.$ |
| 7. $9 a b c.$ | 20. $\sim 23 x y.$ |
| 8. $33 a r x.$ | 21, 22, 23. Answers given. |
| 9. $23 a$ dollars. | 24. $a b.$ |
| 10. $25 a b c.$ | 25. $2 a b x.$ |
| 11. Answer given. | 26. 0. |
| 12. $\sim 23 x y.$ | 27. $\sim 8 a b m x.$ |
| 13. $\sim 50 a b c.$ | 28. $2 a m x.$ |
| 14. $\sim 27 x y z.$ | 29, 30, 31. Answers given. |
| 15. $\sim 23 a b x.$ | |

- | | |
|---------------|--------------|
| 32. $3x.$ | 37. $0.$ |
| 33. $ab.$ | 38. $-3mxy.$ |
| 34. $5bc.$ | 39. $-5ax.$ |
| 35. $-9abcd.$ | 40. $abx.$ |
| 36. $xy.$ | |

SECTION II.

- 1, 2. Answers given in the book.
3. $6x - 5y + 7xy - 4z.$
4. $7ab - 4y + 12bx - 6.$
5. $6abx - 5a - xy + 27.$
6. $19ax - 3am n + xyz - am$
7. $-4y + z + axy - mn.$
8. $ar - 3z - ab - 61.$
- 9, 10. Answers given.
11. $3abc + 6axy - xy.$
12. $ax - ay.$
13. $-2x - 2z + abc.$
14. $x - 3ax + 3y + 159.$
15. $6axy + bmn - 7 + yz.$
16. $x - b.$
17. $2ab + cd.$
18. $5xy - 3.$
19. $6hm - 6.$
20. $-7xyz - mn + 8yz.$
21. $105 + abx + m.$
22. $5xyz - xy.$
23. $4abx - 723.$
24. $3yz.$

SECTION III.

- 1, 2, 3, 4. Answers given in the book.
5. $9 a x - b c + 17 y$.
6. Answer given.
7. $9 (5 a - x y)$.
8. $11 (a b - x + 3 y)$.
9. $9 a (m n - 6 + y z)$.
10. $16 (b c - x y)$.
11. $8 a b + 5 c + 2 a x + 30 - y$.
12. $x + m n$.
13. $15 b c - 13 a x - 26 y + 19 + 4 a c x$.
14. $9 x - 16 y + 18 m + 1 - 2 z$.
15. $4 a b c + 2 a (b - x) - 3 x y z$.
16. $3 a - x y + 2 (x - y) + a (x - y)$.
17. $a b - 1 - m$.
18. $a x y + 10 + 2 a (a - b) - 5 b c$.

CHAPTER III.

SUBTRACTION.

SECTION I.

- | | |
|-------------------------------------|----------------------|
| 1, 2. Answers given
in the book. | 17. $2m - 3n$. |
| 3. $3b$. | 18. $7x - 2y$. |
| 4. $12y$. | 19. $14abc - 2xy$. |
| 5. $7x$. | 20. $12axz - 29hm$. |
| 6. $14abc$. | 21. $19a - 2y$. |
| 7. $3xyz$. | 22. $4z - 5xy$. |
| 8. $11my$. | 23. $x - 13bc$. |
| 9. $2ax$. | 24. $7mx - p$. |
| 10. xyz . | 25. $bc - mn$. |
| 11. 0. | 26. $x - 13bc$. |
| 12. $12hmp$. | 27. $p - 7mx$. |
| 13, 14, 15. Answers
given. | 28. $mn - bc$. |
| 16. $7x - 3y$. | 29. $xyz - xy$. |
| | 30. $32abc - 16$. |
| | 31. $8abcx - 4abc$. |

SECTION II.

- | | |
|---|-----------------------|
| 1, 2, 3. Answers
given in the
book. | 4. $4abc - y$. |
| | 5. $4x - b - z$. |
| | 6. $abcd - 2m - 3y$. |

- | | |
|--------------------------|----------------------------|
| 7. $47 - x - b d.$ | 25. $b c x - x y z + a$ |
| 8. $a b x - 3 a x - 12.$ | 26. $a b - y + x.$ |
| 9. $3 a - b.$ | 27. $64 - x.$ |
| 10. $4 a - 2 x - y.$ | 28. $b x - x z + y.$ |
| 11. $- 16.$ | 29. $a b c - x y z + 9 x.$ |
| 12. $17 - x.$ | 30. $4.$ |
| 13. $b c - 4 y - x.$ | 31. $a x - a b - 7.$ |
| 14. $- x.$ | 32. $x y + 9 a.$ |
| 15. $1 - x.$ | 33. $a d - 9.$ |
| 16. $4 x y - 5.$ | 34. $- m + 5 a.$ |
| 17. $12 - 2 y.$ | 35. $36 - x.$ |
| 18. $x - y.$ | 36. $- b.$ |
| 19, 20. Answers given. | 37. $15 - 4 a.$ |
| 21. $5 x y + a b m.$ | 38. $7 - a.$ |
| 22. $7 - x.$ | 39. $3 (a + b) + x.$ |
| 23. $8 m x + y + 3 a b.$ | 40. $- x (x - y) + x y.$ |
| 24. $24 - b c.$ | 41. $2 a x - b.$ |

SECTION III.

- 1, 2, 3, 4, 5, 6. Answers given in the book.
7. $18 x y - 102 - 59 a b m - 3 a r.$
8. $7 x m - 6 m.$
9. $0.$
10. $35 x y z - 55.$
11. $a n + o p - x y z + 60.$
12. $19 a b x - a m n - 12 b c d - 3 a b c.$
13. $2 y.$
14. $- r y z - 39 + 7 a b + 33 x.$
15. $x + 2 y z - 28 a b c - 7 + x y.$
16. $2 y + 2 z.$

17. $12 a.$

18. $x - 2 z.$

19. $5 a m - x y + z.$

20. $12 m p - 16 a z + 24 - 28 m x y.$

21. $a x + a b.$

22. $3 a m - 35.$

CHAPTER IV.

MULTIPLICATION.

SECTION I.

- | | |
|-------------------------------------|------------------------------------|
| 1, 2, 3. Answers given in the book. | 24. $7 a b c x$. |
| 4. $8 y$ dollars. | 25. $9 a b m y$. |
| 5. $12 a$ bushels. | 26. $7 a b n x y z$. |
| 6. $10 c$ dollars. | 27. $12 a b c r x y$. |
| 7. $7 a$ dollars. | 28. $9 a m n x$. |
| 8. $365 a$ dollars. | 29. $4 a b m y$. |
| 9. $15 x$ yards. | 30. $17 a b c m x y z$. |
| 10. $18 a$ rods. | 31, 32. Answers given in the book. |
| 11. $14 c$ panes. | 33. $32 a x$. |
| 12, 13, 14. Answers given. | 34. $72 a b x$. |
| 15. $b c m n$. | 35. $18 a b c$. |
| 16. $a b x y z$. | 36. $45 a$. |
| 17. $a m n x y z$. | 37. $112 x y z$. |
| 18. $a b c h m n$. | 38. $216 b c m y$. |
| 19. $a b c x$. | 39. $3066 b m y$. |
| 20. $a b m x y z$. | 40. $228 a b x z$. |
| 21. Answer given. | 41. Answer given. |
| 22. $4 a y$ dollars. | 42. $30 a b x$. |
| 23. $4 a b x y$. | 43. $24 a m x y$. |
| | 44. $76 b m x y$. |

45. $24 b x x y z.$

46. $624 m n.$

47. $247 a b x y z.$

48. $14 a m x y.$

49. $1562 a b h m y.$

50. $228 a x y.$

SECTION II.

1, 2, 3. Answers given in the book.

4. $a b b + a b b c + 5 a b x y + 13 a b.$

5. $24 a a b x + 32 a x y + 76 a b x.$

6. $84 a b x y + 56 a x y z + 105 a x y.$

7. $99 a a b m x + 9 a m n x + 9 a m x y.$

8. $a b c x y + a b m x + 4 a b x + a b x z.$

9. $42 a d m m + 7 a m x + 35 a m y + 7 a m.$

10. $5 a a b x x z + a b x + 3 a b x x + a b x y.$

11. $28 b c m n x + 20 b c m y + 32 a b c m.$

12. $9 a x y + a + 6 a a z.$

13. $a b c d + a b b + a b c + a a b + a b x.$

14. $7 a b c d + 9 a b c d x y + 2 e d.$

15, 16. Answers given.

17. $10 a b d y + b b d d + 9 a a y y.$

18. $3 a a b b c c + 8 a a b c y + 4 a b c x + 8 a x y + x x.$

19. $b b + 2 b c + 18 b + c c + 18 c + 80.$

20. $a x + a y + a z + 4 a + c x + c y + c z + 4 c.$

21. $4 a b x y + 2 c x y + x x y y + 4 a b c + c c + 3 a a b b.$

22. $12 a a b + 41 a b + 15 a a + 28 a b b + 14 b b.$

23. $9 a b + 20 + a a b b.$

SECTION III.

1. Answer given in the book.

2. $3 a d x - a b d x.$

3. $8 a a b x - a x.$

The product of $1 \times a x$ is $1 a x$, that is, $a x$.

4. $27 a a c x - 51 a c x.$

5. $a c m n - b c m n x - c m n x y.$

6. $a b x y z + x x y z - x y z + a x y z.$

7. $15 c d x y - 5 b c d x.$

8. $9 a b c m n - 9 m n + 54 m n x.$

9. $a a x x - 4 a x - 4 x - x x.$

10. $a b c c x - 16 a c + b c m x z - 16 m x.$

11. Answer given.

12. $3 a d + b d y - 3 a f - b f y.$

13. $2 a b c + 2 c c + 2 c x - a b d - c d - d x.$

14. $30 a a - 3 a b - 6 b b.$

15. $x x - y y - 2 y z - z z.$

16. $15 x x - 15.$

17. $a b x + c m - 5 a b x y - 5 c m y.$

18. $- 8 r x x - 8 a x - 128 x.$

19. $a a - 10 x - x x - 25.$

20. $b h m + 16 b + b y.$

Observe that $9 b - 8 b$ is equal to b ; all that is required, therefore, is to multiply the given quantity by b .

21. Answer given.

22. $b f + a c f - b c f - b c - a c c + b c c.$

23. $6 a b - a b b - 4 b - 5 a + 20.$

24. $36 a a.$

25. $12 a a b - 8 a - 144 a b + 96.$

26. $48.$

27. $16 a a b b - 46 a b x + 15 x x.$

SECTION IV.

1. Answer given in the book.
2. $xx + 2xy - xxyy + yy.$
3. $3xx + 4xxy - 4x - 4xxyy + 16xy - 15.$
4. $10aa - 27aax + 16ax + 18aaxx - 18aaxx - 8xx.$
5. $4xx + 4bx - 32x - 24bb + 24b + 48.$
6. $21ll - 83lm - 27l + 22mm + 99m.$
7. $aa - bb - 12b - 36.$
8. $abbb - bbcc + 2bccd - ccd d.$
9. $5a + ab - ac + 4b - bb + bc + 45 - 9c.$
10. $28aa - 12ax + 20a + 7ad - 3dx + 5d.$
11. $bbb + ccc.$
12. $6mmm - 2mmx + 2mx - 3mmxy + mx y - xy.$
13. $xxx + xxy + 3xz + xx + xy + 3z.$
14. $a + b - c - ax - bx + cx.$
15. $aaa - 3aax - 3axx + xxx.$
16. $aaa - 3aay + 3a yy - yyy.$
17. $4aaa + 4aac - 4aag + 3acg + 3ccg - 3cgg.$
18. $5mmn - 5mnn + 5mnz - 2myz + 2nyz - 2yz z.$
19. $-16aamy - 3aacy + 64amm + 12acm.$
20. $aaax - bbbx + 8x - aayyy + bbyyy - 8yyy.$
21. $xxx + xyyy - 7x - xxy - yyyy + 7y.$
22. $aa - bb - 2bc - 2bd - cc - 2cd - dd.$
23. $13xyy - 25xy + 12x.$
24. $15aa + 10a - 4ab + 20b - 4bb - 25.$

CHAPTER V.

DIVISION.

SECTION I.

1 to 9. Answers given in the book.	
10.	$2m.$
11.	$9a m.$
12.	$3n p.$
13.	$1.$
14.	$3a.$
15.	$9b h n.$
16.	$2a.$
17.	$4.$
18.	$9c y.$
19.	$3n.$
20.	$8a.$
21.	Answer given.
22.	$\frac{x}{y}.$
23.	$\frac{ab}{cx}.$
24.	$\frac{4abc}{5xy}.$
25.	$\frac{17abx}{21cdy}.$
26.	Answer given.
27.	$\frac{c}{x}.$
28.	$\frac{m}{y}.$
29.	$1.$
30.	$\frac{1}{y}.$
31.	$y.$
32.	$\frac{my}{ax}.$
33.	$\frac{2ab}{c}.$
34.	$\frac{4ab}{x}.$
35.	$\frac{5b}{x}.$
36.	$\frac{3}{a}.$
37.	$\frac{5cm}{2x}.$
38.	$\frac{3}{nx}.$
39.	$\frac{3}{y}.$

10. $\frac{5gh}{3xy}$

11. 7.

12. $\frac{bg}{x}$

13. $\frac{7y}{dm}$

14. $\frac{x}{4y}$

45. $1\frac{1}{2}$.*

46. a .

47. $4b$.

48. $\frac{4}{7}$.

49. $\frac{3}{a}$.

SECTION II.

1, 2, 3. Answers given in the book.

4. $-3hy$

5. $-8b$.

6. -1 .

7. -3 .

8. $-2z$.

9. $-4bn$.

10. $-3h$.

11, 12. Answers given in the book.

13. $-3b$.

14. $-5y$.

15. $-2c$.

16. -1 .

17. -4 .

18. $-7x$.

19. $-8ab$

20. $-4p$.

21, 22. Answers given in the book.

23. $5nx$.

24. $4by$.

25. $5hr$.

26. $4b$.

27. x .

28. $3z$.

29. $4n$.

30. 3.

31. Answer given.

32. $-8x$.

33. $9mn$.

34. $9np$.

35. 7.

36. $3yz$.

37. $-6hm$.

38. $\frac{ab}{cd}$.

* The same quantity ($a - b + 9$), being found in the dividend and the divisor, it is omitted in both; and one coefficient is then divided by the other.

39. $-\frac{a}{b}.$

40. $-\frac{9a}{4}.$

41. $-\frac{9x}{4}.$

42. $\frac{4d}{x}.$

43. $-\frac{3z}{y}.$

44. $-\frac{7m}{3}.$

45. $-\frac{2x}{3}.$

46. $\frac{7b}{3}.$

47. $\frac{7b}{3}.$

48. $-\frac{8}{3}.$

SECTION III.

1, 2. Answers given in the book.

3. The quotient of $18ax$ divided by $3a$ is $6x$;
 the quotient of 15 divided by $3a$ is $\frac{5}{a}$; the
 quotient of $-21ab$ divided by $3a$ is $-7b$.

Ans. $6x + \frac{5}{a} - 7b$.

4. $c + 4x - 3.$

5. $2a - 3.$

6. $r + h - \frac{1}{x}.$

We may represent the division of $-a$ by ax
 thus; $-\frac{a}{ax}$. If we reduce this expression,
 by dividing both terms by a , we obtain $-\frac{1}{x}$,
 as above.

7. $x - y + z.$

8. $2b + 4c.$

9. $5n + 2x.$

10. $2ay + ax - 3m + 4.$

11. $4a - 2 + 3y - 5adx + \frac{m}{4}.$

12. $\frac{3b}{2} - \frac{5bx}{2a} + m.$
13. $4a - 2 - 7y.$
14. $4b - 3c.$
15. $2y - \frac{3y}{x}.$
16. $3b - 3.$
17. $-3b + \frac{2x}{a}.$
18. $1 + \frac{b-8}{a}.$
19. $\frac{3mn}{x} + 7.$
20. $\frac{17}{3a} - 4 + \frac{b}{3}.$
- 21, 22. Answers given.
23. $x.$
24. $a.$
25. $a.$
26. $3a.$
27. $3aa.$
28. $7a.$
- 29, 30, 31, 32. Answers given.

SECTION IV.

1. $a + 5.$
2. $a - 6 + b.$
3. $bb - bc + cc.$
4. $3mmx - mx + x.$
5. $xx + xy + 3z.$
6. $1 - x.$
7. $aa - 4ax + xx.$
8. $aa - 2ay + yy.$

9. $4 a a + 3 c g.$
10. $5 m n - 2 y z.$
11. Answer given.
12. $x + y + \frac{x}{a+b}.$
13. $c + e + \frac{x}{d-h}.$
14. $x x y + \frac{5 y y y}{x-y}.$
15. $b b - a.$
16. $3 a - b + \frac{a x + c}{6 a b + a c c - x x + b b c}.$
17. $- a y + 4 m.$
18. $12 a b - 8.$

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CHAPTER VI.

FRACTIONS.

SECTION I.

1, 2. Answers given in the book.

3. $\frac{6abc}{d}.$

4. $\frac{8a+b-x}{c-d}.$

5. $\frac{17xy-m+7}{x+10}.$

6. $\frac{a-m+7-5y}{10b-x+14}.$

7, 8, 9. Answers given

10. $\frac{24}{1}.$

11. $\frac{a-b}{1}.$

12. $\frac{a+b-mn}{1}.$

13, 14. Answers given.

15. $\frac{40}{5}.$

16. $\frac{ac+bc}{c}.$

17. $\frac{5xz+5yz}{5z}.$

18. $\frac{2a-10+ab-5b}{2+b}.$

$$19. \frac{20ax - 4bcx}{4x}.$$

$$20. \frac{3ax + 3ab - 15a - xz - bz + 5z}{3a - z}.$$

$$21. \frac{30a - 12 + 6y}{6}.$$

$$22. \frac{56a - 2ax + 8ay}{2a}.$$

$$23. \frac{10aabb + 5abx - 25ab}{5ab}.$$

SECTION II.

1, 2, 3, 4. Answers given in the book.

5. $1\frac{3}{8}$.

6. 8.

7. $7\frac{1}{2}$.

8. $15\frac{3}{8}$.

9. 1.

10. $5a$.

11. 1.

12. $3y$.

13. $4b - \frac{bb}{a}$.

14. $aa + ab + bb$.

15. $1 + \frac{2b}{a-b}$.

16, 17. Answers given.

18. $\frac{67}{7}$.

19. $\frac{ab+a}{b}$.

20. $\frac{xyy + 10y + x}{y}$.

21. $\frac{ab+bb+a}{a+b}$.

$$22. \frac{aa - 2ax + xx + a}{a - x}.$$

$$23. \frac{aaaa + bb}{aa}.$$

$$24. \frac{aa - 12a + 12b - bb + x}{a - b}.$$

$$25. \frac{bm - bx + bb + 15m - 15x + 14b}{b + 14}.$$

$$26. \frac{aa + 2ab + bb + a}{a + b}.$$

$$27. \frac{aa - ab + a + b}{a - b}.$$

28, 29, 30. Answers given.

$$31. \frac{45}{135}; \frac{54}{135}; \frac{105}{135}.$$

$$32. \frac{adf}{bdf}, \frac{bcf}{bdf}, \frac{bde}{bdf}.$$

$$33. \frac{15bx}{5ab}, \frac{ay}{5ab}.$$

$$34. \frac{aac}{abc}, \frac{abb}{abc}, \frac{bcc}{abc}.$$

$$35. \frac{15cx}{5ac}, \frac{2ab}{5ac}, \frac{5acy}{5ac}.$$

$$36. \frac{5bdy}{6bdy}, \frac{6ady}{6bdy}, \frac{18bdx}{6bdy}, \frac{6aby + 6bcy}{6bdy}$$

$$37. \frac{2ax}{10x}, \frac{15}{10x}.$$

$$38. \frac{xz}{x}, \frac{3y}{x}, \frac{8x}{x}.$$

39, 40, 41, 42. Answers given.

$$43. \frac{a}{3x}.$$

$$44. \frac{y}{3a + 9y}.$$

$$45. \frac{b - 2x}{3a + 5y}.$$

$$46. \frac{6aa - 3}{4ay + 5}.$$

47. $\frac{2x-1+4a}{3x-5+6xz}$.

48. Since the same quantity is not contained in all the terms of the numerator and denominator, this fraction cannot be reduced.

Ans. $\frac{ax+by-xy}{ay+cy+ax}$.

49. $\frac{b-2x+3y}{4x-2m+b}$

SECTION III.

- 1, 2, 3, 4. Answers given in the book.

5. $\frac{3ac-a-b}{c}$.

6. $\frac{8xy-h+7}{8}$.

7. $\frac{3abm+14-y}{3m}$.

8. $\frac{32abx+5m+7y}{4a}$.

9. $\frac{9am-2x+4y}{a}$.

10. $\frac{xx-xy+a-b}{x-y}$.

SECTION IV.

1. Answer given in the book.

2. $\frac{15a}{c}$.

3. Having added together the several numerators, place the result over the common denominator.

Ans. $\frac{8a-3b}{x-y}$.

4. $\frac{x}{2ab}$.

5. Answer given.

6. $\frac{a+b+c+2d}{x}$.

$$7. \frac{ba - c + c}{x - y}.$$

$$8. \frac{4a}{x + 7 - x}$$

$$9. \frac{2a + 1 - b}{x + y}.$$

10, 11. Answers given.

$$12. \frac{18acd + 16abc + 15ad}{12bcd}.$$

$$13. \frac{4aac + 5abb + 6bcc}{abc}.$$

$$14. \frac{9a + 4b + 15}{20}.$$

$$15. \frac{126a + 16x + 10y}{21}, \text{ or } 6a + \frac{16x + 10y}{21}.$$

$$16. 6x + 5 + \frac{28x + 3ax + 2a}{4a}.$$

If $6x$ and 5 be reduced to the form of fractions, the answer to this question will be expressed in a different manner; but its *value* will be the same.

$$17. \frac{8bx + 11abx + ab + 6ax + 10a + 2aby}{2ab}.$$

$$18. \frac{4bxy - 3abx + 2bx - 4axy - 7xy + bby + aby - bby}{bxy}.$$

$$19. \frac{7abc + 111c + 7x + 7y}{7c}.$$

$$20. \frac{6ad + 6bc + 24bdx + 5bd}{6bd}.$$

21. Answer given.

22. Reduce the fractions to a common denominator, and add together the numerators.

$$\text{Ans. } \frac{33x}{12}, \text{ or } 2\frac{3}{4}x.$$

$$23. \frac{17a}{12}.$$

$$24. \frac{153ab + 40abc}{72}.$$

25. Remember that $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{1}{2}$, are the coefficients of $(a - x)$, which is to be regarded as a simple quantity. These fractions must be added together, and their sum placed before $(a - x)$.

$$\text{Ans. } \frac{23}{12}(a - x), \text{ or } \frac{23a - 23x}{12}.$$

SECTION V.

1. Answer given in the book.

2. $\frac{x-x}{y}$.

3. The fractions are first reduced to a common denominator, and then one numerator is taken from the other.

$$\text{Ans. } \frac{bc-ad}{bd}.$$

4. $\frac{3bcx-20amx}{5xz}$.

5. Reduce the mixed quantities to improper fractions, and these fractions to others having a common denominator.

$$\text{Ans. } \frac{11cy+4a-4b-4cx+7c}{4c}.$$

6. $\frac{25a-5}{8}$.

7. $\frac{2xx-15x-1}{xx-1}$.

8. If these fractions are reduced to a common denominator, we have $\frac{1}{a+b} = \frac{a-b}{aa-bb}$, and $\frac{1}{a-b} = \frac{a+b}{aa-bb}$; the former numerator is then subtracted from the latter.

$$\text{Ans. } \frac{2b}{aa-bb}.$$

9. $\frac{abx-3a+ay-bx-by}{ab}$.
10. $\frac{81a+42+4b}{28}$.
11. $\frac{4x-a-b-12y}{4}$.
12. $y - \frac{x-8xy}{z}$.
13. $\frac{32x+4y-2xz+zz}{4z}$.
14. $\frac{36-aa+mm}{6a+6m}$.
15. $\frac{ax-20x}{5a}$.
16. $\frac{35ax-3x}{7}$.
17. $\frac{ax+bx-a+b}{x}$.
18. $\frac{3x-3y-2a+4}{3}$.
19. $\frac{27x-aa+bb}{a+b}$.

SECTION VI.

1, 2. Answers given
in the book.

3. $\frac{4ay+4by}{7}$.

4. $\frac{ax-2x-2a+4}{b+c}$.

5. $4ab y - 6y$.

6. $4ab - 8aax$.

7, 8, 9. Answers
given.

10. $\frac{3a-b}{3x+2y}$.

11. $\frac{8+a}{3-2x}$.

12. $\frac{a-x}{2-y}$.

13. Answer given.

14. $\frac{3c}{bx}$.

15. $\frac{3ax+3xz}{4y}$.

$$16. \frac{ax - bx}{by - 4y}.$$

$$17. \frac{ay + by - a - b}{3x + xz + 15 + 5z}.$$

$$18. \frac{abc h - 3bc}{4a - 4x}.$$

$$19. 1.$$

$$20. \frac{9aam - 3ax}{2x - 2y}.$$

SECTION VII.

1, 2. Answers given in the book.

$$3. \frac{1}{bc}.$$

$$4. \frac{2x}{b-c}.$$

$$5. \frac{2a}{3xy}.$$

$$6. \frac{2}{a+2x}.$$

7, 8. Answers given.

$$9. \frac{y}{b-c}.$$

$$10. \frac{4-b}{a+8}.$$

$$11. \frac{7ax}{3by - 3bz}.$$

$$12. \frac{2m-x}{3yz}.$$

$$13. \frac{2b-8xy}{x+8}.$$

14, 15, 16. Answers given.

$$17. \frac{3ax - bx - 3ay + by}{bc}.$$

$$18. \frac{8(a-b)}{x(a+b)}.$$

$$19. \frac{32ab(8m-n)}{xyz(a+8)}.$$

$$20. \frac{hy + 4a + 4b + ay}{8x - 8y - ax + ay}.$$

$$21. \frac{x-1}{xz + yz}.$$

$$22. \frac{2ay - 3y}{y-4}.$$

$$23. \frac{bx}{ay}.$$

$$24. \frac{-bx}{-ay}.$$

$$25. \frac{9a}{bx}.$$

$$26. \frac{xx - xy}{aa - ab}.$$

CHAPTER VII.

POWERS.

SECTION I.

- | | |
|--------------------------------|--------------------------------|
| 1. Answer given in the book. | 10. $2401 a^4 b^4 c^4$. |
| 2. $x^6; d^4; c^5; a^8; x^7$. | 11. $7776 x^5 y^5 z^5$. |
| 3. Answer given. | 12, 13, 14. Answers given. |
| 4. $x^4 y^4$. | 15. $a^6 b^2 x^4$. |
| 5. $a^6 b^6 c^6$. | 16. $a^6 b^9 x^3$. |
| 6. $a^9 b^9 x^9$. | 17. $m^{16} x^8 y^{20}$. |
| 7. $m^7 n^7 y^7$. | 18. $36 a^4 c^6 x^8$. |
| 8. Answer given. | 19. $27 x^6 y^9 z^{12}$. |
| 9. $125 a^3 x^3$. | 20. $256 a^8 x^{12} y^8 z^4$. |

SECTION II.

- 1, 2, 3. Answers given in the book.
4. $a^3 + 3 a^2 + 3 a + 1$.
5. $x^2 + 2 x y + 2 x z + y^2 + 2 y z + z^2$.
6. $a^2 + 2 a b + 2 a c + 2 a d + b^2 + 2 b c + 2 b d + c^2 + 2 c d + d^2$.
7. $8 a^3 - 12 a^2 x + 12 a^2 c^2 + 6 a x^2 - 12 a c^2 x + 6 a c^4 - x^3 + 3 c^3 x^2 - 3 c^4 x + c^6$.
8. $27 a^3 + 54 a^2 b + 36 a b^2 + 8 b^3$.

9. $a^2 + 2ab - 2ac + b^2 - 2bc + c^2$.
10. $a^3 - 3a^2x + 24a^2 - 48ax + 3ax^2 + 192a + 24x^2 - 192x - x^3 + 512$
- 11, 12. Answers given.
13. $\frac{9a^4b^4}{x^2y^4}$.
14. $\frac{16a^4b^4x^2}{9a^4y^4}$.
15. $\frac{a^2 + 2ab + b^2}{x^2 - 2xy + y^2}$.
16. $\frac{a^3 - 15a^2 + 75a - 125}{x^3}$.
17. $\frac{27x^6y^2}{8a^3}$.
18. $\frac{4a^2 - 12a + 9}{b^4 + 2b^2c + c^2}$.
19. $\frac{256}{16x^2y^4}$.
20. $\frac{25x^4 - 10a^2x^2 + a^4}{81}$.
21. $\frac{y^2 - 2xy + x^2}{a^4b^2}$.

SECTION III.

- 1, 2, 3. Answers given in the book.
4. $x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$.
5. $a^5 + 5a^4c + 10a^3c^2 + 10a^2c^3 + 5ac^4 + c^5$.
6. $m^8 - 8m^7n + 28m^6n^2 - 56m^5n^3 + 70m^4n^4 - 56m^3n^5 + 28m^2n^6 - 8mn^7 + n^8$.
7. $m^4 + 4m^3n + 6m^2n^2 + 4mn^3 + n^4$.
8. $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$.

9. $a^9 + 9 a^8 b + 36 a^7 b^2 + 84 a^6 b^3 + 126 a^5 b^4 + 126 a^4 b^5 + 84 a^3 b^6 + 36 a^2 b^7 + 9 a b^8 + b^9.$
10. $c^{10} + 10 c^9 d + 45 c^8 d^2 + 120 c^7 d^3 + 210 c^6 d^4 + 252 c^5 d^5 + 210 c^4 d^6 + 120 c^3 d^7 + 45 c^2 d^8 + 10 c d^9 + d^{10}.$
11. $a^{13} - 13 a^{12} b + 78 a^{11} b^2 - 286 a^{10} b^3 + 715 a^9 b^4 - 1287 a^8 b^5 + 1716 a^7 b^6 - 1716 a^6 b^7 + 1287 a^5 b^8 - 715 a^4 b^9 + 286 a^3 b^{10} - 78 a^2 b^{11} + 13 a b^{12} - b^{13}.$
12. $x^6 + 6 x^5 z + 15 x^4 z^2 + 20 x^3 z^3 + 15 x^2 z^4 + 6 x z^5 + z^6.$
13. $c^7 - 7 c^6 d + 21 c^5 d^2 - 35 c^4 d^3 + 35 c^3 d^4 - 21 c^2 d^5 + 7 c d^6 - d^7.$
- 14, 15, 16, 17, 18. Answers given.
19. $a^3 - 6 a b + 9 b^2.$
20. $125 a^6 + 75 a^4 b + 15 a^2 b^3 + b^3.$
21. $9 a^4 + 30 a^2 b^3 + 25 b^6.$
22. $a^2 + 2 a b^2 - 14 a + b^4 - 14 b^2 + 49.$
23. $8 x^3 - 12 x^2 y + 12 x^2 z^2 + 6 x y^2 - 12 x y z^2 + 6 x z^4 - y^3 + 3 y^2 z^2 - 3 y z^4 + z^6.$
24. $a^{10} - 5 a^8 c - 10 a^8 d + 10 a^6 c^2 + 40 a^6 c d + 40 a^6 d^2 - 10 a^4 c^3 - 60 a^4 c^2 d - 120 a^4 c d^2 - 80 a^4 d^3 + 5 a^2 c^4 + 40 a^2 c^3 d + 120 a^2 c^2 d^2 + 160 a^2 c d^3 + 80 a^2 d^4 - c^5 - 10 c^4 d - 40 c^3 d^2 - 80 c^2 d^3 - 80 c d^4 - 32 d^5.$
25. $16807 x^{10} y^{25} - 120050 x^{13} y^{20} z^2 + 343000 x^{16} y^{15} z^4 - 490000 x^{19} y^{10} z^6 + 350000 x^{22} y^5 z^8 - 100000 x^{25} z^{10}.$

SECTION IV.

- 1, 2, 3, 4. Answers given in the book.
5. $a^3 + a^4$.
6. $14 a^2$.
7. $5 a^3 + 6 x^2$.
8. $(a + b + c + d) x^2$.
9. $6 a^3 b - a b^3$.
10. $-a^3 x^2 + b y^2 + d x^2$.
11. $9 a (x^2 - y^2)^2$, or $9 a x^4 - 18 a x^2 y^2 + 9 a y^4$.
12. $12 (a^2 - b^2)^2$, or $12 a^4 - 24 a^2 b^2 + 12 b^4$.
13. $9 a^2 b c^3 + 2 a^2 b^3 c$.
14. $15 a^3 + 10 b c^2$.
15. $4\frac{1}{2} (a^2 - b x)^3$.
16. $9 a^2 b^3 + 15 - x^2$.
17. $14 a^3 (b^2 - 12 a^3 + y^4)^3$.
18. $3 x^2 y^3 + 6 - 6 b^2$.

SECTION V.

- 1, 2. Answers given in the book.
3. $2 (a^3 - b^3)$, or $2 a^2 - 2 b^3$.
4. $a^2 (a^3 - b^3 + 3)^2$.
5. $4 a^5 b^3$.
6. $3 a^3 - 2 a^2$.
7. $2 a^3$.
8. $2 a^3 x - 5 a^2 b$.
9. $9 a^2 y - 7 a^2 x$.
10. $3 a^2 x - 7 a^2 x$.
11. $3 a^2 x y - 3 x^3$.
12. $6 a^2 x^3 - 28 + 32 a^2 b x^3$.

13. $5 a^2 x^3 + 9 x y^2$.
14. $2 (a^2 - b^3)^2 - x^2 y$.
15. $x (a - b)^3$.
16. $2 a^3 (b^2 - 8)^3$.
17. $3 x^2 (b^2 - c^3)$, or $3 b^2 x^2 - 3 c^2 x^2$.
18. $5 a^2 b + 4 x^2 y^3 - 25$.
19. $2 x^3 y (a b^2 - 17)^3$.
20. $2 a b^2 c^3 + 3 b^2 c^3 - 7 a - 3 a^2$.

SECTION VI.

- 1, 2. Answers given in the book.
3. $16 a^5 b^4 x^2$.
4. $15 a^6 b^5 x^4 y^6$.
5. $81 a^6 c^4 m^2$.
6. $x^5 + 5 a x^2$.
7. $6 a^5 x^2 - 10 a^4 x^5$.
8. $a^4 y^3 - 6 a^3 y^2 + a^3 y^5$.
9. $a^6 - b^4$.
10. $2 a^4 b^4 c^3 - 24 a^2 b^3$.
11. $x^4 - y^4$.
12. $2 a^5 + a^4 + 3 a^3 - 11 a^2 - 5 a - 6$.
13. $a^8 - a^2$.
14. Answer given.
15. $a^3 b (x^3 - 12 + x^2)^6$.
16. $15 (a^2 + b^3)^4$.

To find the *value* of this answer, we must multiply the fourth power of $a^2 + b^3$ by 15. The fourth power of $a^2 + b^3$ is $a^8 + 4 a^6 b^3 + 6 a^4 b^6 + 4 a^2 b^9 + b^{12}$; which, when multiplied by 15, becomes $15 a^8 + 60 a^6 b^3 + 90 a^4 b^6 + 60 a^2 b^9 + 15 b^{12}$

In the question, we are required to multiply 3 times the second power of $a^2 + b^3$ by 5 times the second power of the same quantity. Now $(a^2 + b^3)^2 = a^4 + 2 a^2 b^3 + b^6$. If we multiply this by 3, we have $3 a^4 + 6 a^2 b^3 + 3 b^6$; and, if we multiply it by 5, we have $5 a^4 + 10 a^2 b^3 + 5 b^6$. The product of $3 a^4 + 6 a^2 b^3 + 3 b^6$, multiplied by $5 a^4 + 10 a^2 b^3 + 5 b^6$, is $15 a^8 + 60 a^6 b^3 + 90 a^4 b^6 + 60 a^2 b^9 + 15 b^{12}$, which is the value of the answer given above.

Let the student express the answer to question 14th in a similar manner.

17. $6 a (x^3 - y^2 + 4)^6$.
18. $12 a^4 (x^3 - y^3)^4$.
19. $20 a^5 - 88 a^4 x + 47 a^3 x^2 - 6 a^2 x^3$.
20. $a^5 + 32 b^5$.
21. $2 x^2 + 6 a^2 x^2 - b^3 x^2 - 2 a - 6 a^3 + a b^3$.
22.
$$\frac{x^6 - y^4}{a^4 + a^2 b - 8 a^2 - 8 b^4}$$

SECTION VII

- 1, 2. Answers given in the book.
3. $a^3 b$.
4. $b c$.
5. $4 a^5 b$.
6. $3 m y$.
7. Answer given.
8. 1.
9. 1.
- 10, 11. Answers given.

12. $a^{-3} b^{-2} x^{-4}$.

13, 14. Answers given.

15. $(a^2 - x^3 + b)^2$, or $a^4 - 2 a^2 x^3 + 2 a^2 b + x^6 - 2 b x^3 + b^2$.

We obtain the first answer to this question, by subtracting the exponent of the divisor from that of the dividend; all the terms included by the parenthetical marks being taken collectively as a simple quantity. The second answer is of the same value as the first, it being the second power of $a^2 - x^3 + b$.

If we involve the given quantity, $a^2 - x^3 + b$, and divide its fourth power by its second, we shall obtain the same result; but the process will be much longer.

16. $(a^3 + 5 - y)^{-4}$.

17. Answer given.

18. b^3 .

19. $3 a^3 - 5 a^2 b + 2 a b^2$.

20. $x^2 + 4 x - 16$.

21. $b^6 + 2 b^4 c^2 + 4 b^2 c^4 + 8 c^6$.

22. $a^2 - b^2 + 8$.

23. $a^2 - x + \frac{x^4}{a^4 - x^2}$.

CHAPTER VIII.

EQUATIONS OF THE FIRST DEGREE.

SECTION II.

- 1, 2, 3. Answers given in the book.
4. Suppose he gave x cents to the first. Then he gave $2x$ cents to the second, $3x$ cents to the third, and $4x$ cents to the fourth; and to all he gave $x + 2x + 3x + 4x$, that is, $10x$ cents.

Therefore, $10x = 70$.

Ans. To the first, 7 cents; the second, 14; the third, 21; the fourth, 28.

5. The son, 13 years; the father, 65 years.
6. A, \$87; B, \$261; C, \$348.
7. 27 days.
8. Let x = the number of each.

Then $47x$ = the price of all the oxen

$25x$ = the price of all the cows,

and $7x$ = the price of all the sheep.

By the question, $47x + 25x + 7x = 632$.

Ans. 8.

9. A, \$327; B, \$654; C, \$981; D, \$1635.
10. Let x = the price of one apple. Then $2x$ = the price of a pear. And, as the boy gave

twice as much for an orange as for both a pear and an apple, $6x$ = the price of an orange. Then 2 oranges cost $12x$ cents, 3 pears $6x$ cents, and 4 apples $4x$ cents.

Therefore, $12x + 6x + 4x = 22$.

Ans. Apple, 1 cent;

Pear, 2 cents;

Orange, 6 cents.

11. A, 84 years; B, 42 years; C, 14 years.

12. 16 cords.

13. Let x = the number of tons in each lot.

Then $19x + 20x + 22x = 366$.

Each lot contained 6 tons.

Ans. 18 tons.

14. 10 hours.

15. Let x = the ward's share.

Then $x + 4x + 9x + 6x = 60000$.

Ans. The ward, \$3000;

Each daughter, \$6000;

Each son, \$9000;

The widow, \$18000.

16. 43 days.

17. A, \$317; B, \$951; C, \$1268; D, \$2219.

18. Let x = the hours.

The first cock discharges 2 gallons in 15 minutes, or 8 gallons an hour; the second discharges 5 gallons in 30 minutes, or 10 gallons an hour; the third discharges 3 gallons in 45 minutes, or 4 gallons an hour.

Therefore, $8x + 10x + 4x = 264$.

Ans. 12 hours.

19. 14 hours.

20. A, \$9; B, \$3; C, \$6; D, \$12.

21. 23 dollars.

22. Let x = the son's age.

$$\text{Then } 7x - 5x = 16.$$

Ans. 8 years.

23. Let x = the daughter's share. Then $3x$ = the son's share, and $8x$ = the mother's.

$$\text{By the question, } 8x - 3x = 8235.$$

Ans. Daughter, \$1647;

Son, \$4941;

Mother, \$13176.

24. Horse, \$32; chaise, \$128; house, \$640.

25. 20 days.

26. 4 days.

27. 2, 4, 6, 8, 10 and 12 years.

28. 36 gallons.

29. 15 days.

30. Let x = his daily wages.

Then $6x$ = what he earned in one week,

and $2x$ = what he spent in a week.

Ans. 2 dollars.

31. 24 barrels.

32. Let x = the share of one daughter. Then $2x$ = the shares of both daughters, and $4x$ = the shares of the two sons. Now, as the widow is to have one third part of the estate, $4x + 2x$, or $6x$, must be the other two thirds, and $3x$ = the share of the widow.

$$\text{Therefore, } 2x + 4x + 3x = 21546.$$

Ans. A daughter, \$2394;

A son, \$4788;

The widow, \$7182.

33. A, \$9; B, \$27; C, \$18.

SECTION III.

- 1, 2, 3. Answers given in the book.
4. 36 years.
5. 14000 dollars.
6. 39 geese.
7. 24 feet.
8. 480.
9. Let x = the man's estate.
 Then $x - \frac{3x}{5} = 978$.
 Ans. \$2445.
10. 48 years.
11. 441.
12. 120 bushels.
13. 8 dollars.
14. A, \$70; B, \$250.
15. A's, 15 years; B's, 39 years.
16. The first, \$35; the other, \$15.
17. 120 dollars.
18. Let x = the sum he had at first. Then $\frac{3x}{8} =$
 the sum invested; and $\frac{2}{3}$ of $\frac{3x}{8} = \frac{6x}{24}$, or $\frac{x}{4}$, =
 the sum lost.
 Then $\frac{3x}{8} - \frac{x}{4} = 723$, by the question.
 Ans. \$5784.
19. Let x = his age. Then $\frac{9x}{10}$ = the part of his
 life passed; and $\frac{2}{3}$ of $\frac{9x}{10} = \frac{18x}{30}$, or $\frac{3x}{5}$, = the
 years passed in trade. Now, the years he
 was engaged in business, added to his age
 when he commenced, together, give the por-

tion of his life which had been passed when he retired from business.

$$\text{Therefore, } \frac{9x}{10} - \frac{3x}{5} = 21.$$

Ans. 70 years.

20. Let x = the age of B.

Then $\frac{x}{2}$ = the age of A ;

and $\frac{1}{3}$ of $\frac{x}{2}$, or $\frac{x}{6}$, = the age of C.

$$x + \frac{x}{2} + \frac{x}{6} = 120, \text{ by the question.}$$

Ans. A, 36 ; B, 72 ; C, 12 years.

21. 50 bushels of corn ; 30 of oats ; 10 of barley.

22. A, 56 cents ; B, 35 cents ; C, 7 cents.

23. \$45 ; and \$30.

24. Let x = the larger number ;

and $\frac{5x}{7}$ = the smaller number.

$$\text{Then } x + \frac{5x}{7} = 84, \text{ by the question.}$$

Ans. 49 and 35.

25. 28 and 63.

26. A, \$156 ; B, \$234 ; C, \$390.

27. 45, 54, 63, 72.

28. Let $2x$ = the first, $3x$ = the second, and $4x$ = the third.

$$\text{Then } 2x + 3x + 4x = 36.$$

Ans. 8, 12, 16

29. Let x = the distance from A to B.

Then $\frac{3x}{2}$ = the distance from C to D.

Now, $\frac{1}{4}$ of the distance from A to B is $\frac{x}{4}$; and

$\frac{1}{2}$ of the distance from C to D is $\frac{3x}{4}$; there-

fore, $\frac{x}{4} + \frac{3x}{4} = \frac{4x}{4}$, or x , is 3 times the distance from B to C; that is,

$$\frac{x}{3} = \text{the distance from B to C.}$$

$$\text{Then } x + \frac{3x}{2} + \frac{x}{3} = 102.$$

Ans. From A to B, 36 miles;

From B to C, 12 miles;

From C to D, 54 miles.

20. Let $x = A$'s capital. Then $\frac{6x}{7} = B$'s capital.

At the end of the first year, A had $\frac{6x}{5}$, and B had $\frac{5x}{7}$. At the end of the second year, A had $\frac{3x}{4}$, and B had x .

$$\text{Therefore, } \frac{3x}{4} + x = 2450.$$

Ans. A, \$1400; B, \$1200.

31. 43608 dollars.

32. 12 years.

33. 3148 dollars.

SECTION IV.

1. Answer given in the book.

2. A, \$106; B, \$81.

3. A, \$206; B, \$119.

4. Answer given.

5. Let $x = A$'s share. Then $x + 337 = B$'s share; and $x - 181 = C$'s share.

$$x + x + 337 + x - 181 = 3981.$$

Ans. A, \$1275; B, \$1612; C, \$1094.

6. Widow, \$3879; son, \$3287; daughter, \$2765.

7. First, 12; second, 7; third, 20; fourth, 3.

8. Answer given.

9. Let x = the number of hours.

Then $21x$ gallons flowed in, and $12x$ gallons flowed out, while the water was running through both pipes. But 72 gallons had been discharged, before the water began to flow in.

Therefore, $21x = 12x + 72$.

Ans. 8 hours.

10. 26 years.

11. Son, 20 years; father, 60 years.

12. Let x = the number of oranges.

Then $6x - 12 =$ } all the money he had.
Also $5x + 6 =$ }

Consequently, $6x - 12 = 5x + 6$.

Ans. 18 oranges.

13. Answer given.

14. 24 feet.

15. 64 pounds.

16. Man, 48 years; wife, 44 years.

17. Let x = the number of gallons at 13s.

Then $100 - x$ = the gallons at 9s.

And $13x + 9(100 - x) = 12 \times 100$;
or $13x + 900 - 9x = 1200$.

Ans. 75 gallons at 13s.; 25 gallons at 9s.

18. Let x = the gallons at 9s.

Then $10(x + 20) = 9x + (20 \times 13)$;
or $10x + 200 = 9x + 260$.

Ans. 60 gallons.

19. Let x = the gallons at 15s. Then the cost of the whole mixture is $(10 \times 8) + (25 \times 10)$

+ $15x$, or $330 + 15x$; and, by the question, the *value* of the whole mixture is $12(10 + 25 + x)$, or $420 + 12x$.

Therefore, $330 + 15x = 420 + 12x$.

Ans. 30 gallons.

20. Let $x =$ the gallons of water.

Then $10(x + 80) = (35 \times 9) + (45 \times 13)$.

Ans. 10 gallons.

21. 1350 dollars.

22. 84.

23. Let $x =$ the sum that B spends.

Then A spends $2x + 5$.

B has $80 - x$ left;

and A has $80 - 2x - 5$, or $75 - 2x$, left.

Then $\frac{80-x}{2} = 75 - 2x + 13$.

Ans. A, \$69; B, \$32.

24. Answer given.

25. Let $x =$ the greater number.

Then $72 - x =$ the less;

and $x - \frac{216-3x}{4}$ is $\frac{3}{4}$ of the less subtracted from the greater.

Again, $x - 72 + x$, or $2x - 72$, is the excess of the greater above the less.

Subtract this excess from the less, and $72 - x - 2x + 72$, or $144 - 3x$, is the remainder; $\frac{2}{3}$ of which is $\frac{288-6x}{3}$, or $96 - 2x$.

Therefore, $x - \frac{216-3x}{4} = 96 - 2x$.

Ans. 40 and 32.

26. Let $x =$ B's venture.

Then $\frac{3x}{5} =$ A's venture.

$$\frac{3x}{5} + 23 = \text{A's returns,}$$

$$\text{and } x - 23 = \text{B's returns.}$$

Ans. A, \$45; B, \$75.

27. Watch, \$112; chain, \$48.

28. Let x = the time from noon.

Then $12 - x$ = the time till midnight.

$$x + \frac{x}{2} + \frac{x}{3} + \frac{x}{4} + \frac{x}{5} = 12 - x.$$

Ans. 3 h. 39 m. $17\frac{71}{197}$ sec. P. M.

29. 72 and 96.

30. 20.

31. 150 dollars.

32. Let x = the number.

$$\text{Then } x + 16 + \frac{x}{16} = 84, \text{ by the question.}$$

Ans. 64.

SECTION V.

1. 24.

2. 60 years.

3. 120 dollars.

4. 8 and 4.

5. 69 pounds.

6. 20 minutes after 5 o'clock.

7. 9 hours.

8. Answer given.

9. 32 and 16.

10. Widow, \$700; son, \$500; daughter, \$400.

11. Mother, \$4000; a son, \$2000; a daughter,
\$1000.

12. Let x = the value of the estate.

$$\text{Then } \frac{x}{2} - 1000 = \text{A's share};$$

$$\frac{x}{3} - 800 = \text{B's share};$$

$$\frac{x}{4} - 600 = \text{C's share}.$$

$$\text{Therefore, } \frac{x}{2} + \frac{x}{3} + \frac{x}{4} - 2400 = x.$$

Ans. The whole estate, \$28800;

A's share, \$13400;

B's share, \$8800;

C's share, \$6600.

13. Whole amount, \$12000; each of the sons has \$3000.

14. 7 years.

15. 2 and 74.

16. Let x = the weight of the cover.

$$\text{Then } \frac{x+12}{2} = \text{the weight of the second cup};$$

$$\text{and } \frac{x+12}{2} + x = 36, \text{ by the question.}$$

Ans. Cover, 20 ounces; second cup, 16 ounces.

17. 56.

18. 329

19. 20 hours; A, 180 miles; B, 140 miles.

20. Let x = their income.

Then, as A spends $\frac{x}{7}$ more than his income, an-

nually, in 10 years he will be $\frac{10x}{7}$ in debt.

And, since B saves $\frac{x}{5}$ every year, in 10 years

he will save $\frac{10x}{5}$, or $2x$.

$$\text{Therefore, } 2x = \frac{10x}{5} + 160.$$

Ans. \$280.

21. Let x = the bushels of the fourth field.
 Then $7x + 6 = 4x + 69$.
 ANS. 153 bushels.
22. 147 dollars.
23. 18.
24. Let x = the days he was idle.
 Then $24 - x$ = the days he worked;
 and $753 = 1008 - 42x - 9x$.
 ANS. 5 days.
25. A, \$370; B, \$330.
26. A, \$24; B, \$36; C, \$80; D, \$175.
27. Let x = the number.
 Then $4x - 30 = \frac{x}{4} + 30$. ANS. 16.
28. Let x = his age.
 Then $\frac{3x}{8} \times \frac{x}{9} = x$,
 or $3x^2 = 72x$;
 and $3x = 72$, by dividing by x .
 ANS. 24 years.
29. 100 and 150.
30. Let x = the hours he was going down. Then
 $9x$ = the distance *with* the stream: also $6 - x$ = the hours he was returning; and $18 - 3x$ = the distance *against* the stream.
 Therefore, since he rowed the same distance each way,
 $9x = 18 - 3x$.
 ANS. $13\frac{1}{2}$ miles.
31. 85 gallons of wine; 35 gallons of cider.
32. Let x = the whole sum given away. Then $\frac{x}{4} - 3$ = the sum left at the first house. Now, as he divided the whole sum equally among the 6 families, he evidently gave to each $\frac{x}{6}$.

Therefore, $\frac{x}{4} - 3 = \frac{x}{6}$; and $x = 36$.

Ans. 6 shillings.

33. Let $x =$ the length of the post.

Then $x - 4$ expresses the part of it which stands out of the ground; $\frac{5}{6}$ of which, or $\frac{5x-20}{6}$, is immersed at high tide; and the same portion of it appears above the surface at low water.

By the question, $\frac{x}{8}$ appears above the surface of the water at high tide; and 4 feet are in the ground.

$$\text{Therefore, } x = \frac{5x-20}{6} + \frac{x}{8} + 4.$$

And $x = 16$, the length of the post.

At high tide, the top of the post is 2 feet, and, at low tide, 10 feet, above the surface of the water. Of course, the rise of the tide is 8 feet.

34. 325.

35. A, 12; B, 24; C, 18 days.

36. 72 sheep.

37. Let $x =$ the number of bushels.

Then $5x =$ all his money.

$$\text{Also } \frac{7x}{2} + 24 = \text{all his money.}$$

$$\text{Therefore, } 5x = \frac{7x}{2} + 24.$$

Ans. 16 bushels.

38. Let $x =$ A's share.

$$\text{Then } \frac{7x}{3} = \text{B's share;}$$

$$\text{and } \frac{5x}{3} = \text{C's share.}$$

Ans. A, \$192; B, \$448; C, \$320.

39. John, 24; William, 8 marbles.
40. Eggs, 12 cents; apples, 7 cents.
41. A, \$40; B, \$20.
42. Man, 24 years; wife, 20 years.
43. See the solution of question 25, Sec. IV.
- ANS. 126 dollars.

44. Let $x =$ the number of the hound's leaps.

Then $\frac{4}{3}x =$ the hare's leaps.

Now, 2 of the hound's leaps are equal to 3 of the hare's; therefore, x leaps made by the former, are equal to $\frac{3x}{2}$ leaps made by the latter.

Therefore, $\frac{3x}{2} = \frac{4x}{3} + 50$.

Ans. 300 leaps.

- 45. 20 days.**

46. Let x = the number of days in which C could build the wall. Then, in one day, he could build $\frac{1}{x}$ part of it. Now, since A and B can build $\frac{1}{3}$ part of it in a day; and, with the assistance of C, they can build $\frac{1}{2}$ of it; the difference between $\frac{1}{2}$ and $\frac{1}{3}$ is the part of the wall which C alone can build in a day.

Therefore, $\frac{1}{5} - \frac{1}{8} = \frac{1}{x}$.

Ans. 13 $\frac{1}{4}$ days.

47. 3 hours, 56 minutes, 15 seconds.
48. 2 hours, 38 minutes, $19\frac{9}{11}$ seconds.
49. 30 hours.
50. 96 trees.
51. 16 years.
52. 8 years.

53. Let x = the amount of the money.

Then $2 + \frac{x-2}{6}$ = what A takes out;

and $3 + \frac{5x-28}{36}$ = what B takes out.

$2 + \frac{x-2}{6} = 3 + \frac{5x-28}{36}$, by the question.

Ans. \$20.

54. Answer given.

55. 63.

56. Answer given.

57. Let x = their income.

As A saves $\frac{1}{5}$ of his, he spends annually $\frac{4x}{5}$, and

B spends $\frac{4x}{5} + 80$; which, in 4 years, will

amount to $\frac{16x}{5} + 320$; and this, according

to the conditions of the question, exceeds his income for the same time by 220 dollars.

Therefore, $4x + 220 = \frac{16x}{5} + 320$.

Ans. 125 dollars.

58. 160 dollars.

59. 630 dollars.

60. Let x = the first part.

Then $\frac{2x}{3}$ = the second part;

and $\frac{2x}{4}$, or $\frac{x}{2}$ = the third part.

$$x + \frac{2x}{3} + \frac{x}{2} = 26.$$

Ans. 12, 8, 6.

61. $45\frac{1}{2}$ and $10\frac{1}{2}$.

62. Let x = the number of hours.

The first cock will fill $\frac{1}{5}$ of the cistern, the second will fill $\frac{1}{10}$ of it, and the third will empty $\frac{1}{3}$ of it, in an hour. Therefore, $\frac{1}{5} +$

$\frac{1}{10} - \frac{1}{8}$ shows what part of the cistern is filled in an hour.

Again, if all the cocks will fill the cistern in x hours, in one hour they will fill $\frac{1}{x}$ part of it.

$$\text{Therefore, } \frac{1}{5} + \frac{1}{10} - \frac{1}{8} = \frac{1}{x}.$$

Ans. $5\frac{1}{4}$ hours.

63. See explanation of questions 9 and 28, in this Section.

Ans. 16 years.

64. 54 years.

65. Let x = the number.

If x be divided into two equal parts, each of those parts will be $\frac{x}{2}$; and $\frac{x}{2} \times \frac{x}{2} = \frac{x^2}{4}$.

If x be divided into three equal parts, one of the parts will be $\frac{x}{3}$; and $\frac{x}{3} \times \frac{x}{3} \times \frac{x}{3} = \frac{x^3}{27}$.

Therefore, $\frac{x^2}{27} = \frac{x^2}{4}$, by the question;

$$\text{or } 4x^3 = 27x^2.$$

And, if we divide this equation by x^2 , we shall have

$$4x = 27.$$

Ans. $6\frac{3}{4}$.

66. Let x = the number of sheep.

The first company took $\frac{x}{2} + \frac{1}{2}$, and left $\frac{x-1}{2}$.

The second company took $\frac{x-1}{4} + \frac{1}{2}$, and left $\frac{x-3}{4}$.

The third company took $\frac{x-3}{8} + \frac{1}{2}$, and left $\frac{x-7}{8}$.

The fourth company took $\frac{x-7}{8} + \frac{1}{2}$, and left $\frac{x-15}{16}$.

Then $\frac{x-15}{16} = 8$, by the question.

Ans. 143 sheep.

67. First, \$90; second, \$120.

68. 36 and 18.

69. 45 bushels.

70. 48 and 36.

71. Let x = the value of the clothes.

Then $x + 72$ = the whole wages for a year;

and $\frac{7x + 504}{12}$ = the wages for 7 months.

But he received $x + 32$ for 7 months.

$$\text{Therefore, } x + 32 = \frac{7x + 504}{12}.$$

ANS. 24 dollars.

72. Let x = the acres of wheat.

Then $35 - x$ = the acres of rye.

Now, he receives 5 shillings per acre for reaping the rye, $\frac{7}{3}$ of which is $11\frac{2}{3}$; and this, according to the conditions of the question, is 1 shilling more than he receives for reaping an acre of the wheat.

$$\text{Then } 5(35 - x) + 10\frac{2}{3}x = (13 \times 20);$$

$$\text{or } 175 - 5x + 10\frac{2}{3}x = 260.$$

ANS. 15 acres of wheat;
20 acres of rye.

73. Let x = the days it would last the man.

Then, in one day, the man alone would consume $\frac{1}{x}$ part of the sack; and both together consumed $\frac{1}{15}$ of the sack; consequently, the woman alone consumed $\frac{1}{15} - \frac{1}{x}$ per day.

Again, in 6 days, the two consumed $\frac{6}{15}$ of the meal, leaving $\frac{9}{15}$ or $\frac{3}{5}$ of it to be consumed by the woman alone. Now, if she consumi-

ed $\frac{2}{3}$ of the meal in 30 days, she would consume $\frac{1}{30}$ of $\frac{2}{3}$, that is, $\frac{1}{45}$, in one day.

$$\text{Therefore, } \frac{1}{15} - \frac{1}{x} = \frac{1}{50}.$$

Ans. Man, 21 $\frac{1}{2}$ days; woman, 50 days.

74. Let x = the number of women.

$$\text{Then } 4x + 23 = 75.$$

Ans. 18 men; 13 women; 44 children.

75. A, 3427 acres; B, 2878 acres; C, 3865 acres.

76. Let x = his money.

$$\text{Then } \frac{4x + 60}{3} - 45 = x.$$

Ans. 75 dollars.

SECTION VI.

1, 2. Answers given in the book.

3. Let x = the greater number,
and y = the less number.

$$\text{Then } x + y = 120,$$

$$\text{and } 5x - 4y = 150.$$

$$\frac{150 + 4y}{5} = 120 - y, \text{ by comparing the values of } x.$$

Ans. Greater, 70; less, 50.

4. 36 and 24.

5. Let x = the value of the first kind,
and y = the value of the second, per gallon

$$\text{Then } x + y = (15 \times 2), \text{ or } 30.$$

$$\text{Again, } 2x + 3y = 14(2 + 3), \text{ or } 70.$$

Ans. First, 20 shillings;
Second, 10 shillings.

6. Let x = the numerator, and y = the denominator; that is, let $\frac{x}{y}$ = the fraction.

Then $\frac{x+7}{y} = 2$, } by the conditions of the question.
 and $\frac{x}{y+7} = \frac{1}{4}$, }

Ans. $\frac{3}{5}$.

7. 135 and 168.
 8. A, 26 years; B, 46 years.
 9. 24.
 10. Corn, 3 shillings; potatoes, 2 shillings.
 11. A, \$5000; B, \$4800.
 12. A, \$18; B, \$12.
 13. A, 30 apples; B, 10 apples.
 14. 28 and 15.
 15, 16. Answers given.
 17. A, \$12; B, \$18.
 18. First horse, \$150; second horse, \$200
 19. Let x = the number of barrels at \$5,
 and y = the number at \$6.

$$\begin{aligned} \text{Then } 5x + 6y &= 576, \\ \text{and } \frac{25x}{8} + \frac{24y}{7} + 11 &= 353. \end{aligned}$$

Ans. 48 barrels at \$5;
 56 barrels at \$6.

20. Let $\frac{x}{y}$ represent the fraction.

$$\begin{aligned} \text{Then } \frac{2x}{y+7} &= \frac{1}{2}, \\ \text{and } \frac{x+7}{2y} &= 1. \end{aligned}$$

Ans. $\frac{2}{5}$.

21. Length, 55 feet; breadth, 44 feet.
 22. Let x = the length,
 and y = the breadth, of the hall.

Then, by the conditions of the question, $y + 6$ is to $x + 6$ as 6 to 7; and $y - 6$ is to $x - 6$ as 4 to 5.

$$\text{Hence, } \frac{6x+36}{7} = y + 6;$$

$$\text{and } \frac{4x-24}{5} = y - 6.$$

Ans. Length, 36 feet;
Breadth, 30 feet.

23. Let x = the bushels of rye required,
and y = the bushels of barley.

To obtain two equations, we must compare the *quantity* of the mixture with the several parts; and also the *value* of the whole mixture with the respective values of the several parts. Observe, that the wheat is worth 9 six-pences, the rye 7 six-pences, the barley 6 six-pences, and the mixture 8 six-pences, per bushel.

Then $x + y + 86 = 136$, by comparing the quantities.

Also $(86 \times 9) + 7x + 6y = (136 \times 8)$,
or $774 + 7x + 6y = 1088$, by comparing the values.

Ans. Rye, 14 bushels;
Barley, 36 bushels.

24. Let x represent the size of a crate, and y represent the size of a bale.

Then, as 13 crates and 33 bales fill the room once, and as 5 crates and 9 bales fill $\frac{1}{4}$ of the room, we have the following equations:—

$$13x + 33y = 1, \text{ [the capacity of 1 room,]} \\ \text{and } 5x + 9y = \frac{1}{4}.$$

Ans. 24 crates, or 72 bales.

25. Greater, 36; less, 15.
26. Let x = the value of a gold coin,
and y = the value of a silver coin.
Then $24x + 40y = 1$, [that is, the whole debt,]
and $5x + 15y = \frac{1}{4}$.
Ans. 160 silver coins;
32 gold coins.
27. The majority, 820; the minority, 564.
28. An acre contains 160 square rods.
Ans. Length, 16 rods;
Width, 8 rods.
- 29, 30, 31, 32. Answers given.
33. A, \$48; B, \$12.
34. See the solution of question 54, Sec. V. and
of question 9, Sec. VI. in the book.
Ans. 46
35. In the first drawer, \$50; in the second, \$105.
36. A, \$80; B, \$100.
37. Let x = the bushels of barley,
and y = the bushels of oats.
Then $x + 6 = \frac{7y + 42}{6}$, } by the conditions of
and $x - 6 = \frac{6y - 36}{5}$, } the question.
Ans. Barley, 78 bushels;
Oats, 66 bushels.
38. The silver watch, \$24; the gold watch, \$64.
39. Let x = the pages in the book,
and y = the lines on a page.
Then $(x + 12) \times (y + 3) = xy + 744$,
and $(x - 8) \times (y - 4) = xy - 680$.
Ans. 120 pages; 29 lines on a page.
40. A, 238 acres; B, 324 acres.

41. A, \$1300 ; B, \$1800.

42. Let x = the gallons at 8 shillings, and y = the gallons at 10s. 6d., or $10\frac{1}{2}$ shillings.

Then $8x + 10\frac{1}{2}y = 160 \times 6$, or 960.

After the trader had sold 25 gallons from the first pipe, and 16 gallons from the second, he had $x - 25 + y - 16$ gallons left ; which, at the prices he paid for each, amounted to $8(x - 25) + 10\frac{1}{2}(y - 16)$; that is, to $8x + 10\frac{1}{2}y - 368$ shillings.

Again, $x - 25 + y - 16 + 15\frac{3}{4} - 5\frac{1}{4}$, that is, $x + y - 31$ = gallons in the mixture ; and $8(x + y - 31)$, or $8x + 8y - 248$ shillings = the value of the mixture. Therefore,

$$8x + 10\frac{1}{2}y - 368 = 8x + 8y - 248 ;$$

$$\text{or } 8x + 10\frac{1}{2}y - 8x - 8y = 368 - 248.$$

Ans. 57 gallons at 8 shillings ;
48 gallons at 10s. 6d.

Since $+8x$ and $-8x$, in the last equation, balance each other, we can find the value of y directly. By substituting the value of y in either of the above equations, we obtain the value of x .

43. Let x = the number of gallons in the first barrel, and y = the gallons in the second.

Then $x + y = 32$.

After the several changes, there will be $3x - 5y$ gallons in the first barrel, and $6y - 2x$ gallons in the second.

Therefore, $3x - 5y = 6y - 2x$.

Ans. The first, 22 gallons ;
The second, 10 gallons.

SECTION VII.

- 1, 2. Answers given in the book.
 3. Wheat, 7s. ; rye, 5s. ; barley, 3s.
 4. Let x, y, z and $125 - (x + y + z)$ denote the four parts. Then, by the conditions of the question, $x + 4, y - 4, 4z$, and $\frac{125 - x - y - z}{4}$, are all equal to each other.

Therefore $x + 4 = y - 4$, or $y = x + 8$;

also $x + 4 = 4z$, or $z = \frac{x}{4} + 1$;

$$\text{and } x + 4 = \frac{125 - x - y - z}{4},$$

$$\text{or } 4x + 16 = 125 - x - y - z.$$

If we substitute the values of y and z in the last equation, we have

$$4x + 16 = 125 - x - (x + 8) - \left(\frac{x}{4} + 1\right).$$

Ans. 16, 24, 5 and 80.

5. The numbers are 10, 22 and 26.
 6. Let x, y and z denote the number of days in which A, B and C, respectively, can do the work. Then the part of the work, which each can perform in one day, is $\frac{1}{x}, \frac{1}{y}$ and $\frac{1}{z}$.
 Now, by the conditions of the question, A and B can perform $\frac{1}{8}$ part of it, A and C $\frac{1}{9}$, and B and C $\frac{1}{10}$, in a day.

A. Therefore $\frac{1}{x} + \frac{1}{y} = \frac{1}{8}$;

B. $\frac{1}{x} + \frac{1}{z} = \frac{1}{9}$;

C. $\frac{1}{y} + \frac{1}{z} = \frac{1}{10}$.

The value of $\frac{1}{x}$, in equation A, is $\frac{1}{8} - \frac{1}{y}$; and

if this value be substituted for $\frac{1}{x}$, in equation B, we shall have

$$D. \quad \frac{1}{8} - \frac{1}{y} + \frac{1}{x} = \frac{1}{9}.$$

$$C. \quad \frac{1}{y} + \frac{1}{x} = \frac{1}{10}.$$

E. Then $\frac{1}{8} * + \frac{2}{x} = \frac{19}{90}$, by addition;

and $z = 23\frac{7}{11}$, by reducing equation E.

The values of x and y may be found more readily by a similar process, than by substituting the value of z in equations B and C, which the fractions would render tedious. If we subtract C from D, instead of adding these equations, as above, we shall obtain the value of y directly.

Ans. A, $14\frac{34}{11}$; B, $17\frac{4}{11}$; C, $23\frac{7}{11}$ days.

7. A, 20 cents; B, 30 cents; C, 40 cents.

8. The sum divided was 1200 dollars.

A's share, \$500; B's, \$400; C's, \$300.

9. Husband, 40; wife, 36; son, 12 years.

10. Let $\frac{v}{x} =$ the first fraction,

and $\frac{y}{z} =$ the second fraction.

$$\text{Then } \frac{v+3}{x} = \frac{2y}{z},$$

$$\frac{v}{x+3} = \frac{y}{z},$$

$$\frac{v}{x} + \frac{y}{z} = \frac{9v}{x} - \frac{9y}{z},$$

$$\text{and } \frac{vy+10}{xz} = \frac{v}{x}.$$

Ans $\frac{1}{12}$ and $\frac{1}{8}$.

11. If we represent the several prices by u , x , y and z , we shall have

$$7u + 5x + 8y = 121 (7 + 5 + 8),$$

$$3u + 10y + 5z = 150 (3 + 10 + 5),$$

$$8u + 10x + 10y + 7z = 4800,$$

$$18u + 15z = 4800.$$

Ans. A, \$1; B, \$1,20; C, \$1,40; D, \$2.

12. Let $\frac{x}{y}$ and $\frac{z}{y}$ represent the fractions; the value of z being greater than that of x .

A. Then $\frac{x-1}{y} = \frac{z}{3y},$

B. $\frac{z-1}{y} = \frac{2x}{y},$

C. and $\frac{z-x}{y} = \frac{1}{3}.$

By removing the denominators from these equations, we obtain

D. $3xy - 3y = yz$, from A,

E. $zy - y = 2xy$, from B,

F. $3z - 3x = y$, from C.

Divide the equations D and E by y , which is found in all the terms of each.

G. $3x - 3 = z$, or $3x = z + 3$, from D,

H. $z - 1 = 2x$, or $2x = z - 1$, from E.

By subtraction, $x = * \quad 4.$

Substitute the value of x , in equation G: $z = 9.$

Substitute the values of x and z , in equation F:
 $y = 15.$ Ans. $\frac{4}{15}$ and $\frac{9}{15}.$

13. Let $x = A$'s money, $y = B$'s, and $z = C$'s.

Then $x + \frac{y}{2}$, $y + \frac{z}{4}$, and $z + \frac{x}{8}$, are equal to each other; that is, to $\$6\frac{1}{2}.$

Ans. A, \$4; B, \$5; C, \$6.

SECTION VIII.

1. Sherry, \$12; port, \$10 per dozen.
2. Mary, 5 yards; Ann, 7 yards.
3. The numbers are 48 and 60.
4. A, 60 years; B, 36 years.
5. A, \$10; B, \$20; C, \$30.
6. The fraction is $\frac{3}{16}$.
7. A, 46 yards; B, 9 yards; C, 7 yards.
8. Let x = the first, y = the second, and z = the third number. Then, by the conditions of the question,

$$\frac{5x}{6} + \frac{3y}{5} + \frac{2z}{7} = 50,$$

$$\frac{2x}{3} + 2z - 2y = 40,$$

$$\text{and } \frac{5x + 5y + 5z}{12} - 10 = 30.$$

Ans. 24; 30; 42.

9. A, 10 rods; B, 16 rods.
10. Let x , y and z denote the sums he paid to A, B and C, severally, for a week's work.

$$\text{Then } 8x + 8y = 56,$$

$$9x + 9z = 54,$$

$$\text{and } 10y + 10z = 50.$$

Ans. A, \$4; B, \$3; C, \$2.

11. The numbers are 882 and 126.
12. Let x = the bushels of corn, and y = the bushels of oats. Then $5x + 3y$ = the amount of the debt, and $x + y$ = the bushels the chest holds.

After he has delivered 7 bushels of oats, at 3 shillings, there will remain due $5x + 3y -$

21 shillings; for which he must deliver $\frac{5x+3y-21}{5}$ bushels of corn, at 5 shillings.

Now, by the terms of the question, the corn and the oats together do not fill the chest by 2 bushels. Therefore,

$$x + y - 2 = \frac{5x+3y-21}{5} + 7.$$

By a similar course of reasoning, we obtain the equation

$$\frac{5x+3y-36}{3} + 6 = x + y.$$

By reducing these equations, we find the value of x and of y directly; the x 's disappearing in the first equation, and the y 's in the last.

Ans. The debt is \$13½;

The chest holds 21 bushels;

Corn, 9 bushels; oats, 12 bushels.

13. Apples, 2; pears, 3; peaches, 4; oranges, 5

14. A, \$25; B, \$55; C, \$65.

15. Let x = the length of the lot, and y = its width. To find its area, we multiply its length by its width; that is, xy = the number of feet it contained, and $2xy$ = the dollars he paid for it.

Therefore, $2xy = 4350$.

Again, if the lot had been 6 feet wider, its width would have been $y + 6$, and its contents $xy + 6x$ feet; which, at 2 dollars a foot, would have cost $2xy + 12x$ dollars.

Therefore, $2xy + 12x = 5394$.

If the first of these equations be subtracted from the last, the remainder is

$$12x = 1044.$$

Ans. Length, 87 feet; width, 25 feet.

16. In my right hand, 22 cents; in my left, 26 cents.

17. Let x = A's marbles, y = B's, and z = C's.

Then, after the first game, A has $x - y - z$;

B has $2y$; and C has $2z$.

After the second game, A has $2x - 2y - 2z$;

B has $2y - (x - y - z) - 2z$, that is,

$3y - x - z$; and C has $4z$.

After the third game, A has $4x - 4y - 4z$;

B has $6y - 2x - 2z$; and C has $4z -$

$(2x - 2y - 2z) - (3y - x - z)$, that is, $7z - x - y$.

Therefore, by the conditions of the question,

$$\left. \begin{array}{l} 4x - 4y - 4z, \\ 6y - 2x - 2z, \\ 7z - x - y, \end{array} \right\} \begin{array}{l} \text{are equal to each other,} \\ \text{and also to 16.} \end{array}$$

Ans. A, 26; B, 14; C, 8 marbles.

18. Let x = the circumference of the fore-wheel, and y = that of the hind-wheel, in feet.

$$\text{Then } x = \frac{4y}{5},$$

$$\text{and } x + 3 = \frac{5y + 15}{6}.$$

Ans. Fore-wheel, 12 feet;

Hind-wheel, 15 feet.

19. Let x = the number of A's birds, y = the number of B's, and z = the number of C's.

After the first division, A had $x - y - z$; B had $2y$; and C had $2z$.

After the second division, A had $2x - 2y - 2z$; B had $3y - x - z$; and C had $4z$.

After the third division, A had $4x - 4y - 4z$; B had $6y - 2x - 2z$; and C had $7z - x - y$.

Therefore, by the conditions of the question,

$$\left. \begin{array}{l} 4x - 4y - 4z, \\ 6y - 2x - 2z, \\ 7z - x - y, \end{array} \right\} \text{ are equal to each other,} \\ \text{and to } \frac{2}{3}, \text{ or } 32.$$

$$\text{Also, } x + y + z = 96.$$

Ans. A, 52 birds;

B, 28 birds;

C, 16 birds.

20. Let x = the proceeds of A's venture, y = B's, and z = C's. Then, after the several divisions, A must have $\frac{48x - 16y - 16z}{27}$; B, $\frac{52y - 12x - 12z}{27}$; and C, $\frac{55z - 9x - 9y}{27}$.

Therefore, by the conditions of the question,

$$\left. \begin{array}{l} \frac{48x - 16y - 16z}{27}, \\ \frac{52y - 12x - 12z}{27}, \\ \frac{55z - 9x - 9y}{27}, \end{array} \right\} \text{ are equal to each other, and} \\ \text{also to } 64.$$

As these fractions have a common denominator, their numerators are equal to each other.

Ans. A's, \$75; B's, \$63; C's, \$54.

CHAPTER IX.

EXERCISES IN GENERALIZATION

-
1. Answer given in the book.
 2. Twice 90, or 180.
 3. 272.
 4. 750.
 5. Answer given.
 6. Twice 25, or 50.
 7. 124.
 8. 728.
 9. Answer given.
 10. $81 - 49 = 32$.
 11. $144 - 64 = 80$.
 12. $576 - 225 = 351$.
 13. Answer given.
 14. 53603687.
 15. 28995456.
 16. 17519526495.
 17. 11999952.
 - 18, 19, 20, 21. Answers given.
 22. A, \$2125; B, \$1404.
 23. The horse, \$156; the chaise, \$231.
 24. Mother, \$5310; son, \$3824.
 25. A's farm, 87 acres; B's, 75 acres.

26. According to the terms of the question, 16 dollars must be *half* the difference of the two sums; the *whole* difference is, therefore, 32 dollars.
- Ans. A, \$49; B, \$81.
27. One, 95 gallons; the other, 108 gallons
28. Answer given.
29. 435 gallons.
30. 325 pounds.
31. 102 barrels.
32. 37 gallons.
- 33, 34. Answers given.
35. Son, 17 years; father, 68 years.
36. One, \$324; the other, \$2268.
37. A's estate, \$7293; B's estate, \$29172.
- 38, 39, 40. Answers given.
41. The shares are \$2184, \$2912 and \$3640.
42. A, \$1580; B, \$2212.
43. A's loss, \$519; B's, \$865; C's, \$1211.
41. A's share, \$81; B's, \$189; C's, \$243.
45. Answer given.
46. A, \$800; B, \$400; C, \$960.
47. A, \$24; B, \$16; C, \$42.
48. A, \$1200; B, \$900.
49. Answer given.
50. $37\frac{1}{2}$ pounds at 10d.; $74\frac{3}{4}$ pounds at 7d.
51. 12 gallons at a dollar; 24 gallons at 3s.
52. Answer given.
53. B travels 30 hours; A, 40 hours. They travel 120 miles.
54. B travels 9 hours; A, 15 hours. They travel 45 miles.
55. Answer given.

56. The parts are $8\frac{1}{2}$ feet, and $11\frac{1}{2}$ feet.
 57. The parts are 16 inches, and 20 inches.
 58. Answer given.
 59. The greater part, 60; the less, 40.
 60. The parts must be in the ratio of 1 to 3.
 Ans. The less part, 25; the greater part, 75.
 61. Answer given.
 62. 30.
 63. 115.
 64. 4325 dollars.
 65. Answer given.
 66. A man's wages, 6s.; a boy's, 3s.
 67, 68. Answers given.
 69. In this question, $\frac{a}{1+tr} = \frac{392}{1+(2 \times .06)}$, or $\frac{392}{1.12}$.
 Ans. 350 dollars.
 70. 7160 dollars.
 71. Subtract the present worth from the whole amount.
 Ans. 111 dollars.
Remark. According to the usage of merchants, 6 per cent. of the whole sum, or \$117.66, would be allowed as discount.
 72. 575 dollars.
 73. 580 dollars.
 74. Answer given.
 75. 6 per cent.
 76. 5 per cent.
 77. Answer given.
 78. 840 dollars.
 79. 1200 dollars.
 80. Answer given.
 81. 3 years.
 82. 5 years.

CHAPTER X.

EVOLUTION.

SECTION I.

- | | |
|----------------------------|--|
| 1. $\sqrt{x}$. | 6. $x^{\frac{1}{2}}$. |
| 2. $\sqrt[4]{b^3}$. | 7. $y^{\frac{3}{4}}$. |
| 3. $\sqrt[3]{x^2 + y}$. | 8. $(a^2 + x)^{\frac{3}{2}}$. |
| 4. $\sqrt[5]{79}$. | 9. $c^{\frac{1}{m}}$. |
| 5. $\sqrt{a^2 - b + 14}$. | 10. $(x^3 - c^2 + 12)^{\frac{1}{2}}$. |

SECTION II.

- | | |
|-------------------------------|--|
| 1, 2, 3, 4, 5, 6, 7. | 17. $a x^2 \sqrt[3]{9 b^4}$. |
| Answers given
in the book. | 18. $5 a^2 \sqrt{b}$. |
| 8. $8 a^2 b$. | 19. $4 \sqrt[3]{x^4 y^5}$. |
| 9. $3 a b^2 x^3$. | 20. $\frac{3 a}{4 b^2}$. |
| 10. $3 a^2 x y^3$. | 21. $\frac{3 a^2}{2 b^2 c}$. |
| 11. $2 x y^2$. | 22. $\frac{3 a}{b^2}$. |
| 12. $4 a^2 x y^4$. | 23. $\frac{a x^2}{4 b c^2}$. |
| 13. $a x^2 \sqrt{5}$. | 24. $\frac{4 a^2 b}{x y^2 \sqrt{5}}$. |
| 14. $x^2 y^3 \sqrt[3]{7}$. | |
| 15. $6 a b^2 x^4$. | |
| 16. $2 a^2 b^{\frac{3}{2}}$. | |

25. $\frac{2c^{\frac{4}{3}}x}{a^{\frac{1}{2}}\sqrt[3]{9b}}$
 26. $\frac{\sqrt{2b}}{2x\sqrt{y^3}}$
 27. $\pm 5a b^2$
 28. $5a b^2$
 29. $\pm x^2 \sqrt{7y^3}$
 30. $-3x^2$
 31. $\pm 4a^2 b c^3$
 32. $\pm 8a x^2$

33. $-5m^2 n$
 34. $\pm 4x z^2$
 35. $\pm 4x z^2$
 36. $\pm x^2 z \sqrt{17y^3}$
 37. $-a^2 z^3 \sqrt{18y^4}$
 38. $\frac{a^2 b}{c^{\frac{1}{2}}\sqrt{18x}}$
 39. $\frac{a^3 \sqrt{12x^7}}{5y^2 \sqrt[3]{z^4}}$

SECTION III.

- 1, 2, 3. Answers given in the book.
 4. $2x + 3y$
 5. $a^2 - 2ax + x^2$
 6. $x^2 - 2x + 1$
 7. $2x^3 + 3x^2 - x + 1$
 8. $x - 2a + 1$
 9. $x^3 - x^2 + x$
 10. $x^2 + 2y - 2$
 11. $3a^2 b - 5 + 2a^2 b^3$
 12. $2 + a^2 b - 3y^2$
 13. $2a - b + 3c$
 14. $x^3 + 2x^2 + 3x + 4$
 15. $2a^3 - a + 3$
 16. $\frac{a+b}{c-d}$
 17. $\frac{4x+6y}{3a-2c}$
 18. $\frac{5a-9}{6+2x}$

SECTION IV.

- 1, 2. Answers given in the book.
 3. 385.
 4. 1234.
 5. 2345.
 6. 5306.

7. 3041.	13. $5\frac{1}{2}$.
8. Answer given.	14. 5263.
9. $\frac{7}{8}$.	15. $\frac{8}{9}$.
10. Answer given.	16. $31\frac{8}{9}$.
11. $2\frac{1}{3}$.	17. $\frac{3}{4}$.
12. $2\frac{2}{3}$.	18. 60309.

SECTION V.

1. Answer given in the book.
2. 2.236 &c.
3. 1.414 &c.
4. 28.687 &c.
5. 22.956 &c.
- 6, 7. Answers given.
8. $\frac{4}{5}$, or $1\frac{1}{5}$, &c.

Remark. The 8, 9, 10, 11, and 16, questions admit of a great variety of answers, depending upon the numbers by which the given fractions are multiplied.

9. $2\frac{4}{11}$.
10. $\frac{7}{8}$.
11. $1\frac{1}{3}$.
12. Answer given.
13. $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128}$ &c.
14. $x - \frac{x^2}{2x} - \frac{x^4}{8x^3} - \frac{x^6}{16x^5}$ &c.
15. $a^2 + \frac{1}{2a^2} - \frac{1}{8a^6} + \frac{1}{16a^{10}}$ &c.
16. $\frac{3}{4}$.
17. 87.412 &c.

CHAPTER XI.

EQUATIONS OF THE SECOND DEGREE.

SECTION I.

- 1, 2, 3, 4, 5, 6. Answers given in the book.
7. 12 oranges, at 3 cents.
8. Let x = the number of barrels.

Then $\frac{4x}{5}$ = the price of a barrel;

and $\frac{4x^2}{5}$ = the value of the whole quantity.

Therefore, by the conditions of the question,

$$\frac{5x^2}{4} = \frac{4x^2}{5} + 45.$$

Ans. 10 barrels, at \$8.

9. Let x = one side of the square field.

Then x^2 = its contents.

To find the area or superficial contents of any surface, as of a field or a floor, we multiply its length by its breadth. Therefore, 81×64 , or 5184 , = the contents of the field exchanged. And, by the conditions of the question,

$$x^2 = 5184.$$

Ans. 72 rods.

10. Length, 36 rods; breadth, 24 rods.
11. Let x = the width of the street.

Then $\frac{144}{x} =$ the length.

And $\frac{144}{x^2} = 16$, by the question.

Ans. Length, 48 rods
Width, 3 rods.

12. Answer given.

13. Let $x =$ the son's part,
and $y =$ the daughter's part.

Then $x + y = 14$; and $x = 14 - y$.

$$\text{Also } \frac{9x}{16y} = \frac{y}{x},$$

$$\text{or } 9x^2 = 16y^2.$$

By extracting the square root of each member of this equation, we have

$$3x = 4y,$$

$$\text{and } x = \frac{4y}{3}.$$

Therefore, $\frac{4y}{3} = 14 - y$, by comparing the values of x .

Ans. Son, \$8;
Daughter, \$6.

14. Let x and y denote the sides of the rooms.

Then $\frac{9x^2}{25} = y^2$,
and $x = y + 10$, } by the question.

Ans. Larger room, 25 feet square;
Smaller room, 15 feet.

15. Answer given.

16. *Remark.* A line of soldiers standing abreast, or side by side, is called a *rank*; and a *file* is a row of soldiers standing one behind another, from front to rear.

Let $x =$ the men in a rank.

Then $x + 9 =$ the men in file ;
and $x^2 + 9x =$ the whole number of men.

When the army was formed in 9 lines, each rank was increased by 900 men ; therefore, $(x + 900) \times 9$, or $9x + 8100$, = the whole number of men also.

$$\text{Then } x^2 + 9x = 9x + 8100.$$

Ans. 8910 men.

17. The greater, 18 yards ; the less, 14 yards.

18. 11 years.

19. 7.

20. Length, 30 rods ; breadth, 25 rods.

21. Let $x =$ the length of the entry.

$$\text{Then } \frac{108}{x} = \text{its breadth.}$$

Therefore, by the conditions of the question ;

$$2x - \frac{216}{x} = x + \frac{108}{x}.$$

Ans. 18 feet long ; 6 feet wide.

22. 2 and 6.

23. 25 feet long ; 16 feet wide.

24. Let $x =$ the number of men.

Then $\frac{5x}{4} =$ the number of women, and $x +$

$\frac{5x}{4}$, or $\frac{9x}{4} =$ the persons relieved. There-

fore, $\frac{3x}{4} =$ the shillings given to each man,

and $\frac{3x^2}{4} =$ what all the men received.

Again, $\frac{5x}{4} - x$, or $\frac{x}{4} =$ excess of women above

the men. Therefore, $\frac{x}{2} =$ the shillings given

to each woman, and $\frac{5x^2}{8} =$ what all the women received.

Then $\frac{3x^2}{4} = \frac{5x^2}{8} + 18$, by the question.

Ans. 12 men; 15 women.

25. 6 and 8 years.

26. Length, 8 rods; width, 3 rods.

SECTION II.

1, 2, 3, 4. Answers given in the book.

5. 7.

6. 8.

7, 8. Answers given.

9. 7 and 15.

10. 24 and 6.

11, 12, 13, 14. Answers given.

15. 6.

16. 3.

17. 14 and 5.

18. Length, 25 feet; width, 16 feet.

SECTION III.

1. Man's age, 24 years; woman's, 18 years.

2. Son's age, 19 years; daughter's, 14 years.

3. Width, 24 feet; length, 32 feet.

4. 11 and 17.

5. \$16; \$12.

6. Let x = the length of the garden.

Then, as the whole wall is 128 yards long, the wall on one side and one end must be $\frac{1}{2}$, or 64 yards. Consequently,

64 — x = the width of the garden,
and $64 x - x^2$ = its area.

Therefore, $64 x - x^2 = 1008$.

Ans. Length, 36 yards;

Width, 28 yards.

7. Let x = the number of sheep.

Then $\frac{80}{x}$ = the price of one sheep.

If he had bought 4 sheep more for the same money, the number would have been $x + 4$, and the price of each, $\frac{80}{x+4}$.

Therefore, $\frac{80}{x+4} + 1 = \frac{80}{x}$, by the question.

Ans. 16 sheep.

8. 6 and 8.

9. Watch, \$160; chain, \$40.

10. Let x = the number of silver coins.

Then $24 - x$ = the number of copper coins.

Now, since each silver coin is, by the question, worth $24 - x$ pence, the value of all the silver coins is $24 x - x^2$ pence. Also, as each copper coin is worth x pence, all the copper coins are worth $24 x - x^2$ pence. Of course, the value of all the coins, both silver and copper, is $48 x - 2 x^2$ pence.

Therefore, $48 x - 2 x^2 = 216$; that is, 18×12 .

By changing all the signs in this equation, and dividing the several terms by 2, we obtain

$$x^2 - 24 x = -108.$$

By completing the square.

$$x^2 - 24 x + 144 = -108 + 144, \text{ or } 36,$$

$$x - 12 = \mp 6, \text{ by evolution,}$$

$$\text{and } x = \mp 6 + 12, \text{ that is, } 18, \text{ or } 6.$$

If the root of 36 be a *positive* quantity, there are 18 silver coins, and 6 copper coins. But if the root of 36 be a *negative* quantity, there are 6 silver coins, and 18 copper coins. Either of these answers will meet the conditions of the question. If the silver coins be more valuable than those of copper, they must be less in number.

Ans. 6 silver coins;
18 copper coins.

11. 15 oxen.

12. Let x = the miles B travels per hour.

Then $x + 3$ = the miles A travels.

Again, $\frac{150}{x}$ = the hours B travels,

and $\frac{150}{x+3}$ = the hours A travels.

Now, as A gains 8 hours and 20 minutes of B, that is, $8\frac{1}{3}$ hours, we have the equation,

$$\frac{150}{x+3} + 8\frac{1}{3} = \frac{150}{x}.$$

Ans. B, 6 miles;
A, 9 miles.

13. The numbers are 16 and 9.

14. A's, 32 years; B's, 20 years.

15. The numbers are 21 and 9.

16. Let x = A's age.

Then $\frac{120}{x}$ = B's age.

If A were 3 years younger, his age would be $x - 3$; and if B were 2 years older, his age would be $\frac{120}{x} + 2$. By the conditions of the question, if we multiply $x - 3$ by $\frac{120}{x} + 2$, we have

$$\frac{114x + 2x^2 - 360}{x} = 120.$$

Ans. A's age, 15 years; B's, 8 years

17. 40 sheep.

18. Let x = the number relieved by B.

Then $x + 40$ = the persons relieved by A.

Again, $\frac{1200}{x}$ = the sum given by B to each,

and $\frac{1200}{x + 40}$ = the sum given by A to each.

$$\text{Therefore, } \frac{1200}{x} = \frac{1200}{x + 40} + 5.$$

Ans. A relieved 120 persons,
B, 80 persons.

19. Let x = the yards in the finer piece.

Then $x + 2$ = the yards in the coarser piece.

The finer piece cost £18, or 360 shillings; and
the coarser, £16, or 320 shillings. There-
fore, the fine cost $\frac{360}{x}$ shillings a yard; and

the coarse cost $\frac{320}{x + 2}$ shillings.

$$\text{Then } \frac{320}{x + 2} + 4 = \frac{360}{x}, \text{ by the question.}$$

Ans. 18 yards of the fine, at 20s.

20 yards of the coarse, at 16s.

20. See the solution of the 16th question, Chap.
XI. Sec. I.

Ans. 75 men in rank; 16 men in file.

21. 36 rows of 25 trees each.

22. Let x = the length of the room. Then, as the
whole perimeter of the room is 48 feet, one
side and one end must measure 24 feet; and
 $24 - x$ = its width. If we subtract $24 - x$
from x , we have $2x - 24$, which is the dif-
ference of its length and breadth.

Therefore, $35 (2x - 24) = 24x - x^2$.

Ans. 14 feet long; 10 feet wide.

23. Let x = the number of sheep bought. Then $x - 8$ = the number sold. Now, as the whole cost 190 dollars, or 1140 shillings, $\frac{1140}{x}$ = the price paid for a sheep, and $\frac{1140}{x} + 8$ = the price to be received. But $\frac{1140}{x+8}$ = the price to be received also.

$$\text{Therefore, } \frac{1140}{x} + 8 = \frac{1140}{x+8}.$$

Ans. 38 sheep, at \$5 each.

24. £40.

25. 30 dollars.

26. 75 calves, at 16 shillings each.

27. Let x = the width of the border.

Then the area of the four portions of the border, marked 18, 12, 18, 12, in the annexed diagram, will be $18x + 12x + 18x + 12x$ or $60x$; and the area of the four little square at the corners, will be $x^2 + x^2 + x^2 + x^2$ or $4x^2$.

x	18	x
12	$18 \times 12 = 216,$ area of grass-plot.	12
x	18	x

Therefore, the whole area of the border is $4x^2 + 60x$; and, by the conditions of the question,

$$4x^2 + 60x = 216,$$

or $x^2 + 15x = 54$, by dividing by 4.

Ans. 3 yards.

28. Let x = the width of the walk. Then $6x - 2$ = one side of the court, and $24x - 8$ = its whole periphery. Also $24x^2 - 8x + 4x^2$, or $28x^2 - 8x$, is the area of the walk. Therefore, by the terms of the question,

$$28x^2 - 8x = 24x - 8 + 164,$$

$$\text{or } 28x^2 - 32x = 156,$$

and $x = 3$, the width of the walk.

Hence, $6x - 2 = 18 - 2$, or 16 yards, the side of the square.

Ans. 256 yards.

29. The number is 6.

30. 49.

31. Boy's age, 17 years; girl's, 11 years.

32. The smaller, 26 yards; the larger, 38 yards.

33. Let x = the number of rods.

Then $\frac{105}{x}$ = the days he was employed.

Therefore, $(x - 2) \times \left(\frac{105}{x} + 6\right) = 105$.

Ans. 7 rods.

34. Let x = the price of a calf. Then $\frac{10}{x}$ = the number of calves sold for \$10; and $\frac{10}{x} - 3$, or $\frac{10-3x}{x}$ = the sheep sold for \$6. To find the price of one sheep, we must divide \$6

by $\frac{10-3x}{x}$; that is, $\frac{6x}{10-3x} =$ the price of a sheep.

$$\text{Therefore, } 7x + \frac{72x}{10-3x} = 50.$$

To obtain the answer, we must use the *negative* root of the known quantity; as the other value of x will not meet the conditions of the question.

Ans. Calves, \$2; sheep, \$3.

35. 16 years.
36. 12 barrels.
37. Length, 15 rods; breadth, 10 rods.
38. A, 14 dollars; B, 5 dollars.
39. A's age, 19 years; B's, 9 years; C's, 5 years.
40. Let $x =$ the price of a bushel of wheat,
and $y =$ the price of a bushel of rye.

$$\text{Then } 80x + 100y = (65 \times 20).$$

Again, $\frac{200}{x} =$ the bushels of wheat for £10,

and $\frac{200}{x} + 60 =$ the bushels of rye for £20.

Also, $\frac{400}{y} =$ the bushels of rye sold for £20.

$$\text{Therefore, } \frac{200}{x} + 60 = \frac{400}{y}.$$

Ans. Wheat, 10s. a bushel;
Rye, 5s. a bushel.

CHAPTER XII.

MISCELLANEOUS QUESTIONS.

-
1. 6 dozen.
 2. 9 children ; 18 women ; 54 men.
 3. A's share, \$2654 ; B's, \$6635.
 4. The mother's share, \$7416 ; son's, \$4635 ;
daughter's, \$2781.
 5. The number is 24.
 6. For a sheep, \$5 ; cow, \$30 ; ox, \$45 ; horse,
\$135.
 7. 174 dollars.
 8. 18 dollars ; 28 dollars.
 9. A, 27 acres ; B, 72 acres ; C, 99 acres.
 10. 47 gallons.
 11. 72.
 12. 135 dollars.
 13. Academical department, 288 ; Law, 16 ; Di-
vinity, 32 ; Medicine, 48.
 14. 8 jugs.
 15. A, \$270 ; B, \$360 ; C, \$540.
 16. A, \$144 ; B, \$240 ; C, \$210.
 17. 275 dollars.
 18. The number is 27.
 19. The parts are 8, 27 and 84.

20. Observe that, to take away the cipher from the right of the last number, is the same thing as to divide it by 10.

Ans. 24 dollars per month.

21. 12 travellers; 55 dollars.

22. Let x = the greater number, and y = the less.
Then $x + y$ = the sum of the numbers,
and $x - y$ = their difference.

Therefore, $x^2 + xy = 144$, by the question,

$$\text{and } y = \frac{144 - x^2}{x}.$$

Also, $xy - y^2 = 14$, by the question.

By substituting the value of y and y^2 in this equation, we have

$$144 - x^2 - \frac{20736 - 288x^2 + x^4}{x^2} = 14.$$

If we now complete the square, and extract the root, we obtain

$$2x^2 - 209 = \mp 47.$$

We regard ∓ 47 as a negative quantity.

Ans. 9 and 7.

23. 12 days.

24. Length, 10 rods; width, 8 rods.

25. A, 6 miles; B, 5 miles.

26. Let x = the number of birds killed by the second shot.

Then $2x^2$ = the whole number of birds, and

$$\frac{16x^2}{9} = \text{the number killed by the first shot.}$$

Therefore, $2x^2 = x + \frac{16x^2}{9} + 2$, by the question.

Ans. 72 birds.

27. 36 rods.

28. A, 645 acres; B, 839 acres; C, 1174 acres.

29. The numbers are 6 and 4.

30. 576 men ; 289 men.

31. 6 miles an hour.

32. Let x = the miles A travelled daily. Then $\frac{216}{x}$ = the days they travelled ; which, by the question, is also equal to the miles which A travelled daily more than B. Consequently,
 $x - \frac{216}{x}$ = the miles B travelled daily.

Also $\left(x - \frac{216}{x}\right) \times \frac{215}{x}$ = the miles B travelled.

Therefore, $\frac{216x^2 - 46656}{x^2} + 216 = 396$.

Ans. A, 36 miles ; B, 30 miles.

33. A's share, \$701,92 ; B's share, \$1228,36.

34. Let x and y denote the digits.

Then $10x + y$ = the number of acres.

Also, $x + y + 7 = 3x$,

and $10x + y - 18 = 10y + x$.

Ans. 53 acres.

35. This question produces a pure equation of the *third* degree : of course, the *cube* root of x and of the known quantity must be found.

Ans. 12 gentlemen.

36. A, \$12 ; B, \$16 ; C, \$35 ; D, \$18.

37. The number is 93.

38. The first horse, \$90 ; the second horse, \$60.

39. The numbers are 24, 48 and 60.

40. £1 12s. ; 14 beggars.

41. Suppose he paid away x guineas and y crowns.

Then, by reducing the whole to shillings, we have

$21x + 5y = 297$, the amount paid away,

and $5x + 21y = 249$, the amount left.

Ans. 21 guineas ; 21 crowns.

42. 18 half-guineas ; 72 crowns.
 43. He was idle 6 days.
 44. 75 in rank ; 16 in file.
 45. The larger cock, 9 minutes ; the other, 12.
 46. 33 gallons at 13s. 6d. ; 71 gallons at 9s. 6d.
 47. 6 yards, at \$8 per yard.
 48. Let x = the length of B's garden, and y = its width. Then $x y$ = its area. Also, $\frac{2x}{3}$ = the length of A's garden, and $\frac{40x}{3}$ = its area. Since $\frac{40x}{3}$ is, by the question, equal to $\frac{2}{5}$ of B's garden, we have

$$x y = \frac{200x}{9}. \quad \text{Ans. } 22\frac{2}{3} \text{ yards.}$$

49. Let x = the number of eagles, and y = the number of dollars. Then $2x + \frac{3x}{8} + y$ = the number of coins, and $10x + \frac{5x}{2} + \frac{15x}{8} + y$ = their value in dollars ; one fifth of which is $2x + \frac{x}{2} + \frac{3x}{8} + \frac{y}{5}$. Therefore,

$$2x + \frac{3x}{8} + y = 2x + \frac{x}{2} + \frac{3x}{8} + \frac{y}{5}.$$

$$\text{Also, } x + y = x + \frac{3x}{16} + \frac{y}{2} + 2.$$

Having found the number of coins of each sort, reduce them all to dollars.

Ans. 240 dollars.

50. 48 bushels of barley ; 72 bushels of oats.
 51. 18 men ; 12 women.
 52. Let x = the number of hours required. Then $x + 10$ = the hours the criminal travels. Also, let y = the miles the criminal travels in an hour.

Then $y + 3 =$ the miles the pursuers travel.

Therefore, $x y + 10 y = x y + 3 x$.

Again, at the moment the express met the criminal, the pursuers had travelled $5\frac{2}{5}$ hours, and the criminal $15\frac{2}{5}$ hours, or $\frac{78 y}{5}$ miles; and the pursuers will arrive at the place where the criminal was met, in $2\frac{2}{5}$ hours after they meet the express, that is, in $10\frac{2}{5}$ hours from the commencement of the pursuit; and in $10\frac{2}{5}$ hours they will travel $\frac{52 y + 156}{5}$ miles.

$$\text{Therefore, } \frac{78 y}{5} = \frac{52 y + 156}{5}.$$

Ans. 20 hours.

53. $28.284 +$ rods.

54. Length, 50 rods; width, 32 rods.

55. The dimensions of the field must first be found.

If $x =$ the length of the field, $\frac{5 x}{3} =$ its width,
 $\frac{5 x^2}{8} =$ its contents in rods, and $\frac{5 x^2}{1280}$, or $\frac{x^2}{256} =$
 the number of acres it contained.

Then, as the man paid as many dollars per acre as there were rods in the length of the field, he gave for the whole $\frac{x^3}{256}$ dollars.

Again, the number of rods round the field was $2 x + \frac{10 x}{8}$, which, multiplied by 13, becomes $26 x + \frac{130 x}{8}$.

$$\text{Therefore, } 26 x + \frac{130 x}{8} = \frac{x^3}{256}.$$

Divide all the terms of this equation by x , and you obtain a pure equation of the second degree; which gives 104 rods for the length of

the field, and 65 rods for its width.

Ans. \$4394.

56. This question produces a pure equation of the third degree.

Ans. 5 children.

57. 14 hours.

58. Let x = the number of members. Then, if they pay 80 cents each, the sum raised will be $80x$ cents, which is 18 dollars more than they want. Therefore, the price of the pump is $80x - 1800$ cents.

But, by the other conditions of the question, the price of the pump is $50x + 1200$ cents. Therefore, $80x - 1800 = 50x + 1200$.

Ans. \$62.

59. 30 dozen.

60. 1975 trees.

61. The first, 49; the second, 63; the third, 72 gallons.

62. The first, 28; the second, 9; the third, 15 gallons.

63. A, 24 miles; B, 32 miles.

64. Let x = the less side,
and y = the difference of the sides.

Then $x + y$ = the greater side.

Therefore, $\frac{xy + y^2}{x} = 24$, $\left\{ \begin{array}{l} \text{by the conditions of} \\ \text{the question.} \end{array} \right.$
and $\frac{xy}{x + y} = 6$,

Ans. Length, 24 rods;
Breadth, 12 rods.

65. Let x = the length of the room.

Then $\frac{5x}{6}$ = its width.

$$x^2 - \frac{25x^2}{36} = 891, \text{ by the question.}$$

Ans. Length, 54 feet ;

Width, 45 feet.

66. Let x = the circumference of the fore wheel,
and y = the circumference of the hind wheel.

$$\begin{aligned} \text{Then } \frac{120}{x} &= \frac{120}{y} + 6, \\ \text{also } \frac{120}{x+1} &= \frac{120}{y+1} + 4, \end{aligned} \left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} \text{by the conditions of the} \\ \text{question.} \end{array}$$

Ans. Fore wheel, 4 yards ;

Hind wheel, 5 yards.

67. 36 men ; 16 women. In all, 52 passengers.

68. Let x = the value of A's venture.

$$\text{Then } \frac{1100}{x} = \text{A's gain per cent.}$$

$$\text{Also, } \frac{1100}{x} = \text{B's venture.}$$

$$\text{Again, } \frac{3600x}{1100}, \text{ or } \frac{36x}{11} = \text{B's gain per cent.}$$

$$\text{Therefore, } \frac{144x}{11} = \frac{1100}{x}.$$

Ans. A's venture, \$94 ;

B's venture, \$120.

69. Let x = one of the fractions,
and y = the other.

$$\text{A. Then } xy = \frac{1}{8}, \quad \left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} \text{by the conditions of} \\ \text{the question.} \end{array}$$

$$\text{B. and } x^2 + y^2 = \frac{17}{36}, \quad \left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} \text{the question.} \\ \end{array}$$

$$\text{c. } 2xy = \frac{2}{3}, \text{ by multiplying A by 2.}$$

$$\text{D. } x^2 + 2xy + y^2 = \frac{25}{36}, \text{ by adding c to B,}$$

$$\text{E. and } x + y = \frac{5}{6}, \text{ by evolution.}$$

$$\text{F. Again, } x^2 - 2xy + y^2 = \frac{9}{36}, \text{ by subtracting c from B,}$$

$$\text{G. and } x - y = \frac{3}{6}, \text{ by evolution.}$$

$$2x = \frac{4}{3}, \text{ by adding G to E,}$$

$$\text{and } x = \frac{2}{3}.$$

Also, $2y = \frac{2}{3}$, by subtracting $\frac{1}{3}$ from $\frac{5}{3}$,
and $y = \frac{1}{3}$.

Ans. $\frac{2}{3}$ and $\frac{1}{3}$.

70. 14 persons; 5s. each.

71. Let x = the length of the lot,
and y = the width.

Then $2x + 2y$ = the perimeter,
 $20x + 20y$ = the cost of the land,
and xy = the area in feet.

Also $\sqrt{xy}$ = the side of a square lot of the same
area,

$4\sqrt{xy}$ = its perimeter,
and $40\sqrt{xy}$ = its price at \$10 per foot.
Therefore, $20x + 20y = 40\sqrt{xy} + 330$;

$$\text{that is, } \frac{20x + 20y - 330}{40} = \sqrt{xy},$$

$$\text{or } \frac{2x + 2y - 33}{4} = \sqrt{xy},$$

$$\text{and } \left(\frac{2x + 2y - 33}{4}\right)^2 = xy.$$

Again, $\frac{2x + 2y}{4}$ = side of a square of the same
perimeter,

$$\text{and } \frac{4x^2 + 8xy + 4y^2}{16} = \text{its area.}$$

In $12\frac{1}{4}$ square rods there are $\frac{53361}{16}$ square feet.

$$\text{Therefore, } \frac{4x^2 + 8xy + 4y^2}{16} = xy + \frac{53361}{16}.$$

Ans. Width, 9 rods;
Length, 16 rods.

END.

